

# VOLUMETRIC COMPUTATIONS THROUGH DIGITAL TERRAIN MODELING

*A Thesis Submitted  
In Partial Fulfilment of the Requirements  
for the Degree of  
MASTER OF TECHNOLOGY*

105971

*by*

D. V. NAGARAJU

*to the*

DEPARTMENT OF CIVIL ENGINEERING  
INDIAN INSTITUTE OF TECHNOLOGY, KANPUR

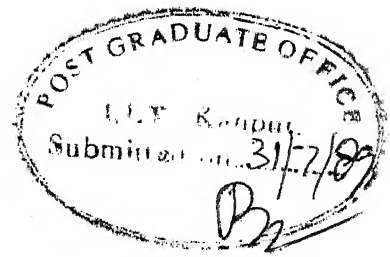
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### CERTIFICATE

This is to certify that the thesis entitled "VOLUMETRIC COMPUTATIONS THROUGH DIGITAL TERRAIN MODELING " submitted by Mr.DINDIGAL VIJAY NAGARAJU, in partial fulfilment of requirements for the degree of 'Master of Technology ' at Indian Institute of Technology, Kanpur, is a record of bonafide work carried out by him under my supervision and guidance. The work presented in this thesis has not been submitted elsewhere for a degree.

July ,1989

  
(Dr.K.K.RAMPAL)  
Professor  
Department of Civil Engg .,  
Indian Institute of Technology  
KANPUR

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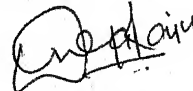
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### ABSTRACT

The quality and effectiveness of decision making relates directly to the quality of data , analytical tools available and sophistication of mathematical model adopted by decision-makers. A close description of Earth's relief is needed for several purposes by administrators, planners and Resource scientists. Such a requirement cannot be met in normal working, except by way of supplementation through various means. Digital Terrain Model (DTM) defined as registration of array of coordinates is one such means. The accuracy of DTM depends on density versus sophistication of mathematical model  $Z = F(x,y)$  approximating the surface . In this investigation, an attempt has been made to evolve a unique system of DTM for volumetric computations.

In Civil Engineering Projects Volumetric computations play a unique role. Particularly in highway design phase, such determinations are essential for both alignment and cost estimation. Volumetric determination through DTM offers an advantage to designer to compare several alignments without recourse to further measurements. Apart from the academic value, this work will have great use in practical methods for planning and execution of Civil Engineering projects.

## CHAPTER I

## INTRODUCTION

1.1 Introduction

The word Photogrammetry is derived from Greek words 'Photos', 'gramma' and 'metron', meaning 'light', 'that which is drawn or written' and 'to measure' respectively. The word 'photogrammetry' literally means measuring from photographs.

The last few years have seen a change in photogrammetry from graphic products to digital, brought out by ubiquitous computers. Another change is on the horizon now -- Digital Terrain Modelling (DTM), which need structured data. "Digital Terrain Modelling is self explanatory -- a DTM is the digital representation of the terrain i.e., a representation of ground surface in terms of X,Y,Z coordinates. X and Y define planimetric position of a terrain point, Z its height. The various uses of DTM include (1) Production and maintenance of cartographic data base (2) Orthophoto production (Orthophoto is photograph showing images of objects in their true or orthographic positions) (3) Engineering planning such as highways, railroads and power lines etc., (4) Simulation (5) Meteorology and Navigation.

The data can be organized as a matrix array of coordinate triplets or as equations of surface defined by polynomials or Fourier series. Characteristics other than elevation such as land value, soil type, depth to bedrock and land use etc., may

also be included in DTM. When only elevation is considered it is normally known as Digital Elevation Models.

### 1.2 Methods of Surface Representation

The data for digital terrain model can be stored as an array shown in Fig.1.1. The grid spacing in X and Y direction remains constant and only elevation (Z-coordinate) of the data points are recorded. Knowing the grid spacing and the number of rows and columns in the grid exact X and Y coordinates of each elevation point can be easily derived. However, since the placement of data points is completely independent of surface features, this method of data distribution may result in the omission of critical surface features. The accuracy of surface representation will depend largely on grid size and surface roughness.

The second method of representation of terrain is in the form of a polynomial. Earth's surface may be represented by 3, 4, 8, 12 or 16 parameter bicubic polynomial in X,Y coordinates. A 16-parameter polynomial could be

$$Z = A + BY + CY^2 + DY^3 + EX + FXY + GXY^2 + HXY^3 + IX^2 + JX^2Y + KX^2Y^2 + LX^2Y^3 + MX^3 + NX^3Y + PX^3Y^2 + QX^3Y^3$$

where A,B,...,P,Q are 16 parameters. The accuracy of surface depends on density of data points collected, sophistication of the mathematical model  $Z = f(X,Y)$  and type of terrain. This is shown fig 1.2.

Truly speaking for complex surfaces the continuous function

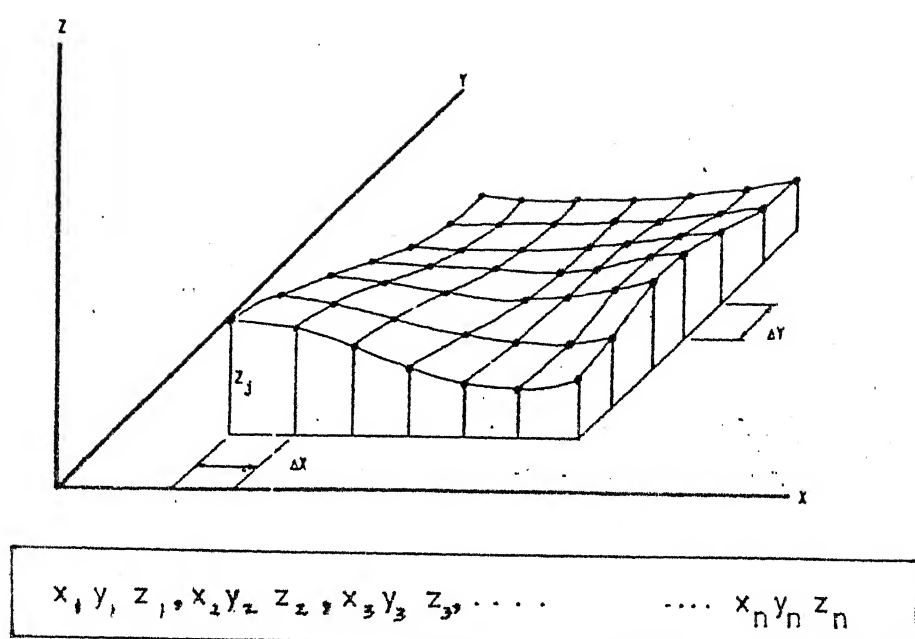


Fig 11 SURFACE REPRESENTATION IN GRIDED PATTERN

FIG 12 SURFACE REPRESENTATION BY POLYNOMIAL

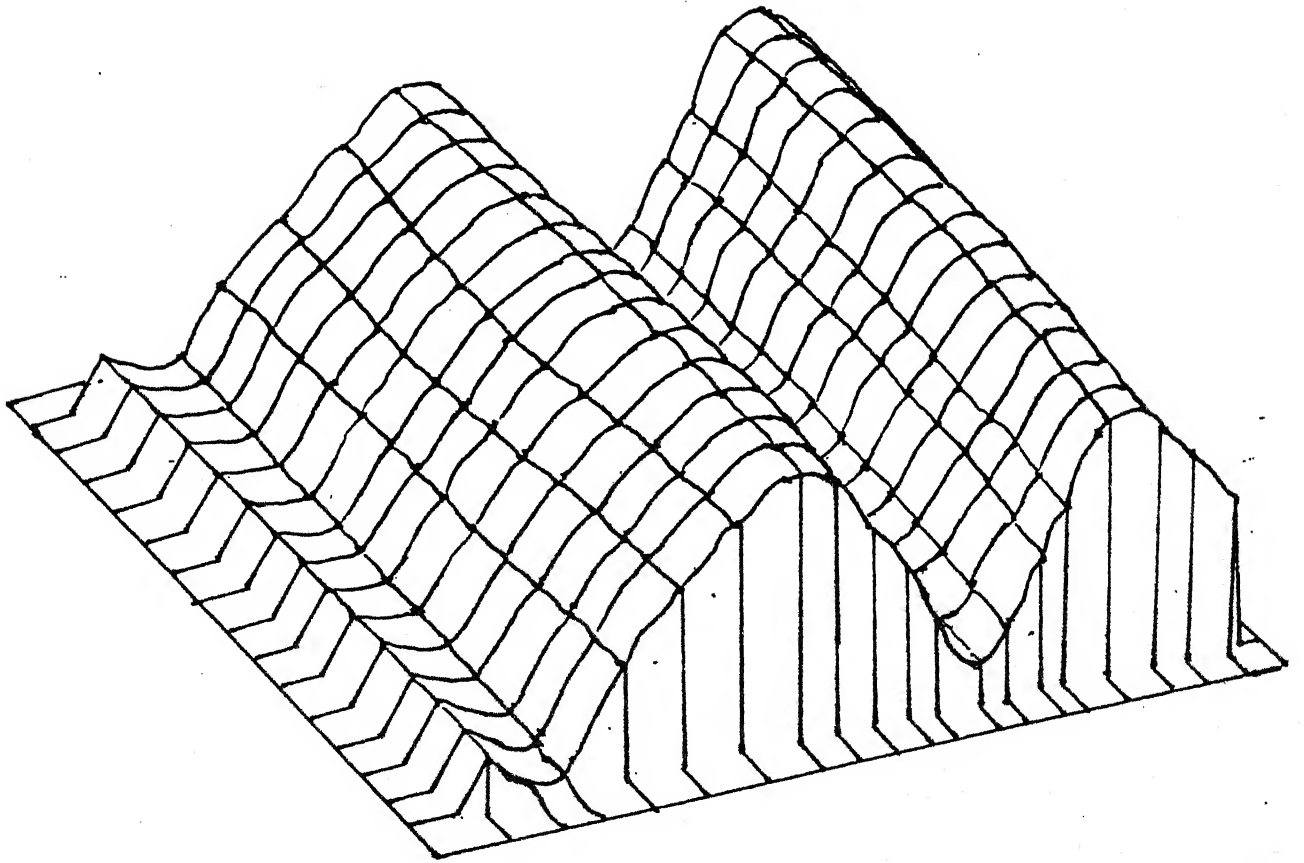


Fig13 SURFACE REPRESENTATION BY FOURIER SERIES.

governing the surface will be a polynomial of the type

$$Z = \sum a_n x^n y^n.$$

Surface representation using Fourier series is a very advanced method of approximating the surface. The terminology associated with Fourier series is derived largely from Electrical Engineering and time series analysis.

In the simplest example, all harmonics in one direction have zero amplitude, and only one amplitude in other direction has an amplitude greater than zero. The resulting surface resembles of a sheet of corrugated iron roofing or a sheet of parallel ripples, or a still body of water as shown in Fig.1.3.

The mathematical background of Fourier series is excellently given by John C. Davis [6].

### 1.3 Need and Objective of Work

Earth's relief is needed for diverse purposes as in contouring on maps, determination of intervisibility of points, terrain simulation, slope analysis, Earth work calculations and various Engineering applications.

Computation of volumes for highways from conventional methods is tedious and expensive. Alternative alignment selection through ground survey measurements is expensive and time consuming process. A rectangular DTM generated by any technique is useful for the above objectives.

The objective of the thesis is to describe

- (1) various methods of data collection
- (2) to explain the method to generate DTM for flat terrain

and undulated terrain .

- (3) to discuss the method of progressive sampling to generate cartographic data base
- (4) to develop a computer program for highway volume calculation, given a DTM and design parameters of highway.

#### 1.4 Organization of the Thesis

Previous works on DTM are discussed in Chapter II. Chapter III describes the various sources of data collection for DTM. Chapter IV is devoted to mathematical formulation and Chapter V describes volume calculation methods. Results are summarized in Chapter VI. Chapter VII sums up the conclusions drawn from the present study and offers certain suggestions for further study.

## CHAPTER II

### LITERATURE REVIEW

This Chapter is devoted to a survey of existing literature on DTM. A review of the existing DTMs outside India and the work done in India in this realm are presented here. The literature available on the application of DTM to volumetric computations is very limited. But, in this thesis the literature available on data acquisition, interpolation methods and applications are interrelated to evolve a unique system of DTM for volumetric computations.

#### 2.1 Introduction

DTM was the brain child of Charles L. Miller at Massachusetts Institute of Technology ca 1955. The objective was to expedite highway design by digital computation based upon photogrammetrically acquired terrain data. Instead of measuring levels along cross sections of a given alignment, X,Y,Z coordinates of points in the band of interest are measured ("band of interest is a strip of ground in which it is known that the final horizontal alignment of road will pass"). The design of highway can take place with the aid of computer. The parameters for any alignment

may be fed in to the computer, volumes, mass movements etc., can be computed by each version which enables the designer to compare several alignments without recourse to further measurements.

## 2.2 Brief Description of some existing DTMs

### 2.2.1 French DTM -- "semis des points"

Sampling pattern	: morphologic points and lines
Interpolation methods	: moving average - to calculate the height Z of a point with known X,Y, a square is defined around this point. A polynomial of second degree is fitted using least squares. The function value of the computed paraboloid at (X, Y) is taken as the height of this point.
Application	: highway design

### 2.2.2. Finnish DTM

Sampling pattern	: regular triangular grid.
Interpolation method	: linear interpolation.
Application	: highway design

### 2.2.3. Hertfordshire County Council DTM

Sampling pattern	: Contours
Interpolation	: linear interpolation
Application	: highway design

### 2.2.4. Czechoslovakian DTM

Sampling pattern	: depends on the type of terrain
Interpolation	: Moving averages
Application	: highway design

The work of Gogia, Survey of India, Chandigarh [5] reveals that for flat terrain least squares technique can be used to generate DTM. He suggested that bilinear interpolation can be adopted for undulated/hilly terrains.

Naitani, in his work, "Photogrammetric methods for Rail route surveys and Profiling" [10] used 1:10000 maps and second order photogrammetric equipments, viz., wild B-8 for mapping. He suggested for corridor area study DTM is much useful. But this aspect was not discussed in his paper.

Makarvic, in his paper, "Progressive sampling for digital terrain models" [8] stated that the method of progressive

sampling of discrete points arranged in a regular grid of varying intensity provides a powerful tool for acquisition of digital terrain model data.

Ghazalli [ 4 ] has done extensive investigation of the performance of progressive sampling. He suggested numerical values of threshold based on nature of the terrain.

Volumetric computation may be carried out using one of the two methods.

(1) Crosssection method

(2) The unit area or borrow pit

#### The Crosssection method

When Crosssection are used ground profiles are taken 10-50 meters normal to present center line. The formula that is commonly used is end area formula .

In this end area formula, volume is assumed to be equal to average area of the two ends multiplied by the distance between them.

$$V = (A_1 + A_2) L/2$$

where  $A_1$  and  $A_2$  are areas of two end bases and  $L$  is the distance between them. Because of its simplicity this formula is almost universally used in computing earthwork.

This is illustrated in fig 2.1.

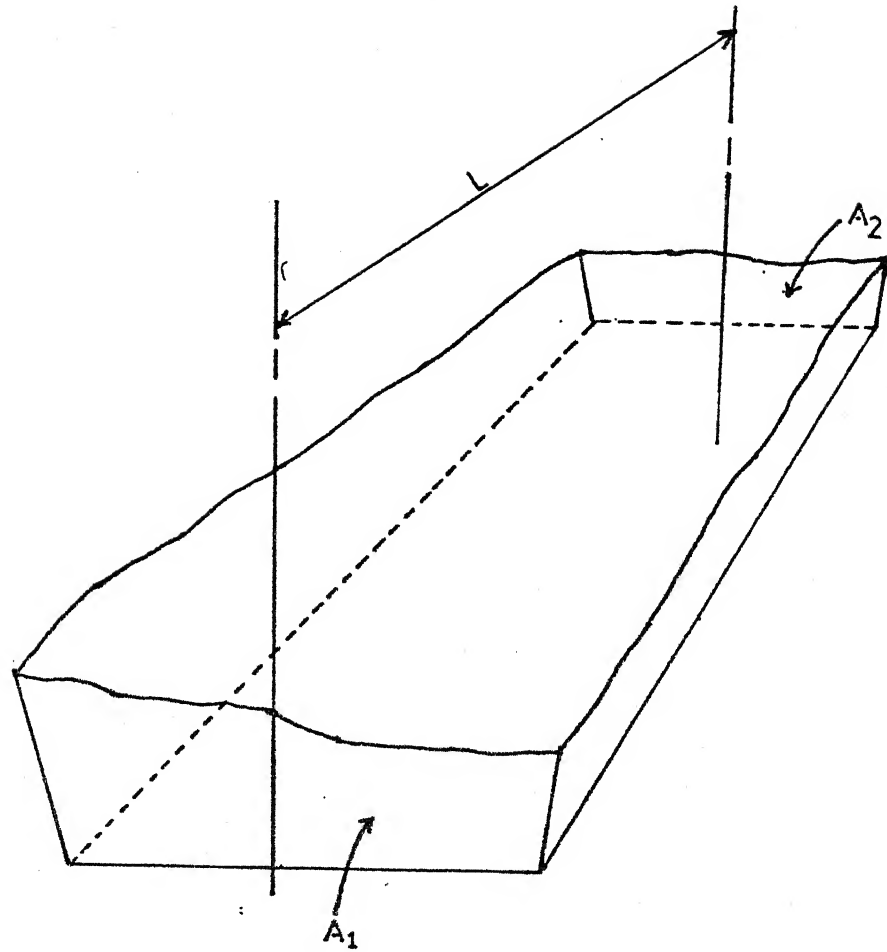


Fig 2.1 AVERAGE. END AREA METHOD.

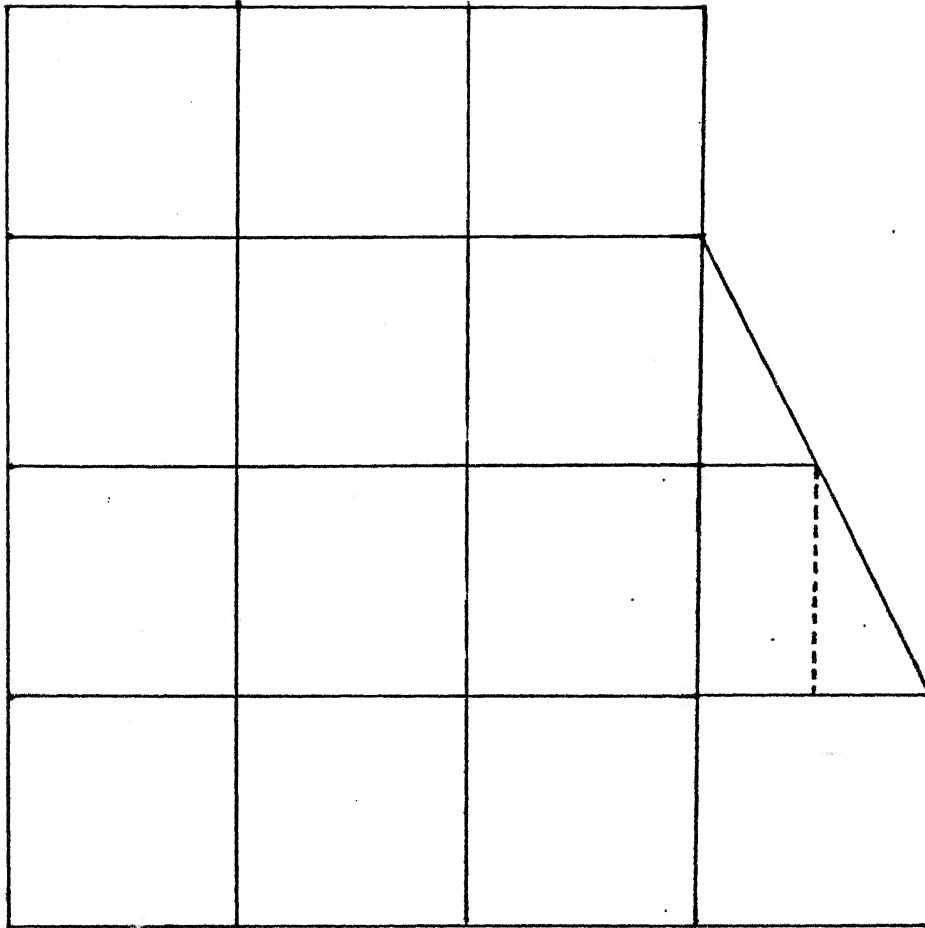


Fig 2.2 BARROW PIT METHOD.

### Borrow PIT method

When the work covers a broad area rather than a narrow strip, as is the case in grading for a large building or industrial yard, or when a borrow pit is not adjacent to the route survey, it is frequently desirable to lay out a system of rectangles (usually squares) say 50 ft or 100 ft on a side depending on the roughness and topography of the ground and precision with which quantities are required.

The volume of a square or a rectangular truncated prism can be calculated by

$$V = A (h_1 + h_2 + h_3 + h_4)/4$$

A is the area of the right section

$h_{1-4}$  are cuts at the four corners of the prism.

Similarly, volume of the truncated triangular prism is calculated using the formula

$$V = A(h_1 + h_2 + h_3)/3$$

This method is illustrated in Fig.2.3.

### 2.3. Proposed Method

From the available literature, to achieve the objectives of the thesis the following method is proposed.

- (1) Data is collected from maps
- (2) Using weighted average method, the data is arranged into grids

- (3) For further densification, least square interpolation is used for flat terrain and bilinear interpolation is used for undulated terrain .
- (4) Progressive sampling is done to locate new points
- (5) For volume calculations cross section method is used.  
This is schematically shown in fig2.3.

A series of computer programs coded in Fortran IV and implemented on DECSYSTEM 1090, based on the above approach, makes it possible to achieve the objectives of the thesis.

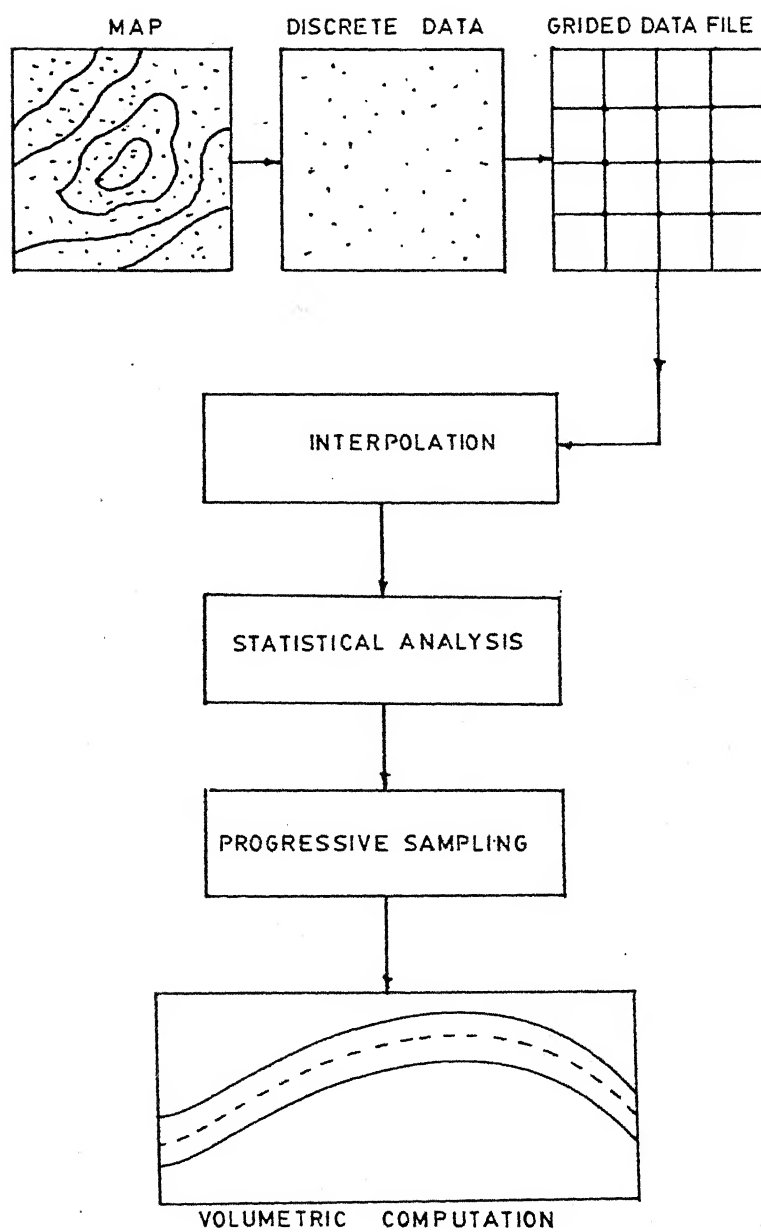


Fig 2.3 SCHEMATIC DIAGRAM OF PROPOSED WORK.

## CHAPTER III

## DATA ACQUISITION

The various methods of data acquisition for digital terrain models are described in this Chapter.

3.1 Sources of Data Collection

There are three possibilities to obtain X,Y,Z coordinates of terrain points.

- (a) Ground Survey
- (b) Photogrammetric Models
- (c) Topographic Maps

3.1.1 Ground Survey

In ground survey tacheometry or leveling is adopted to collect the data. Its a tedious and time consuming task.

3.1.2 Photogrammetric Models

These are commonly used to collect data. The data can be collected using stereo plotters. Stereoscopic plotting instruments (commonly called stereo plotters or simply plotters) are instruments designed to provide rigorously accurate analog solutions for object positions from their corresponding image positions on overlapping pairs of photos. In aerial photography, the photographs are taken in such a way that there exists 60% overlap in longitudinal direction and 20% in transverse direction as shown in Fig.3.1. To form a three-dimensional model of the

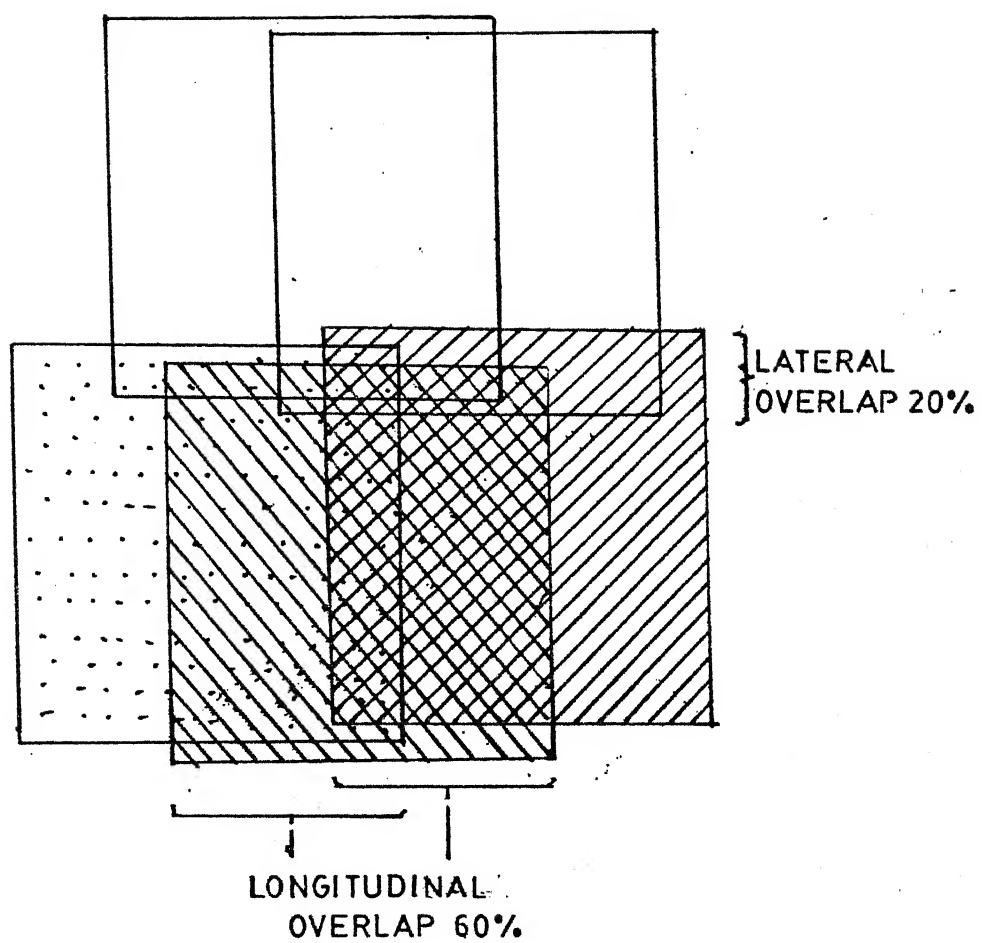


Fig3.1 OVERLAPPING PHOTOGRAPHS.

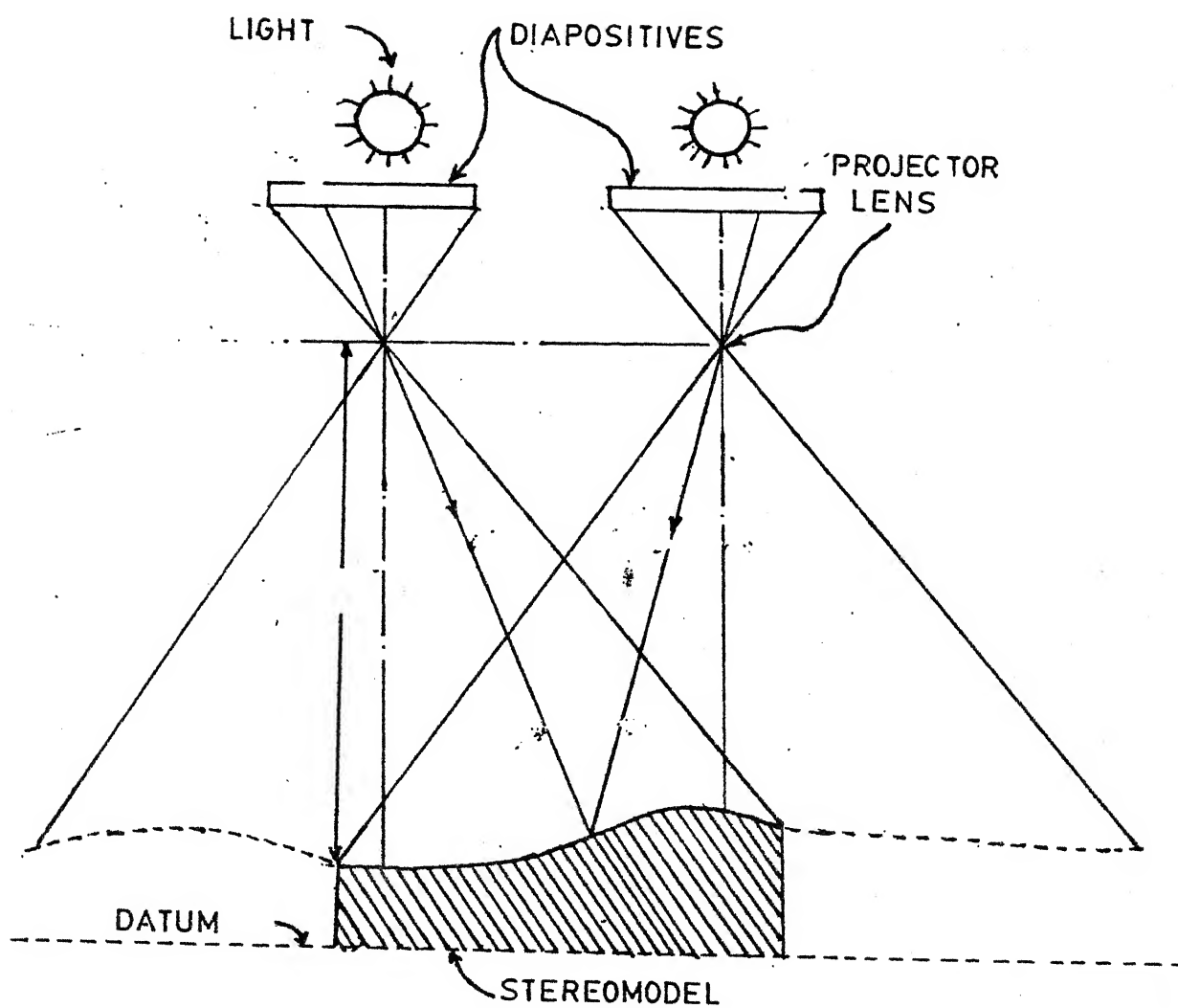


Fig 3.2 INTERIOR ORIENTATION .

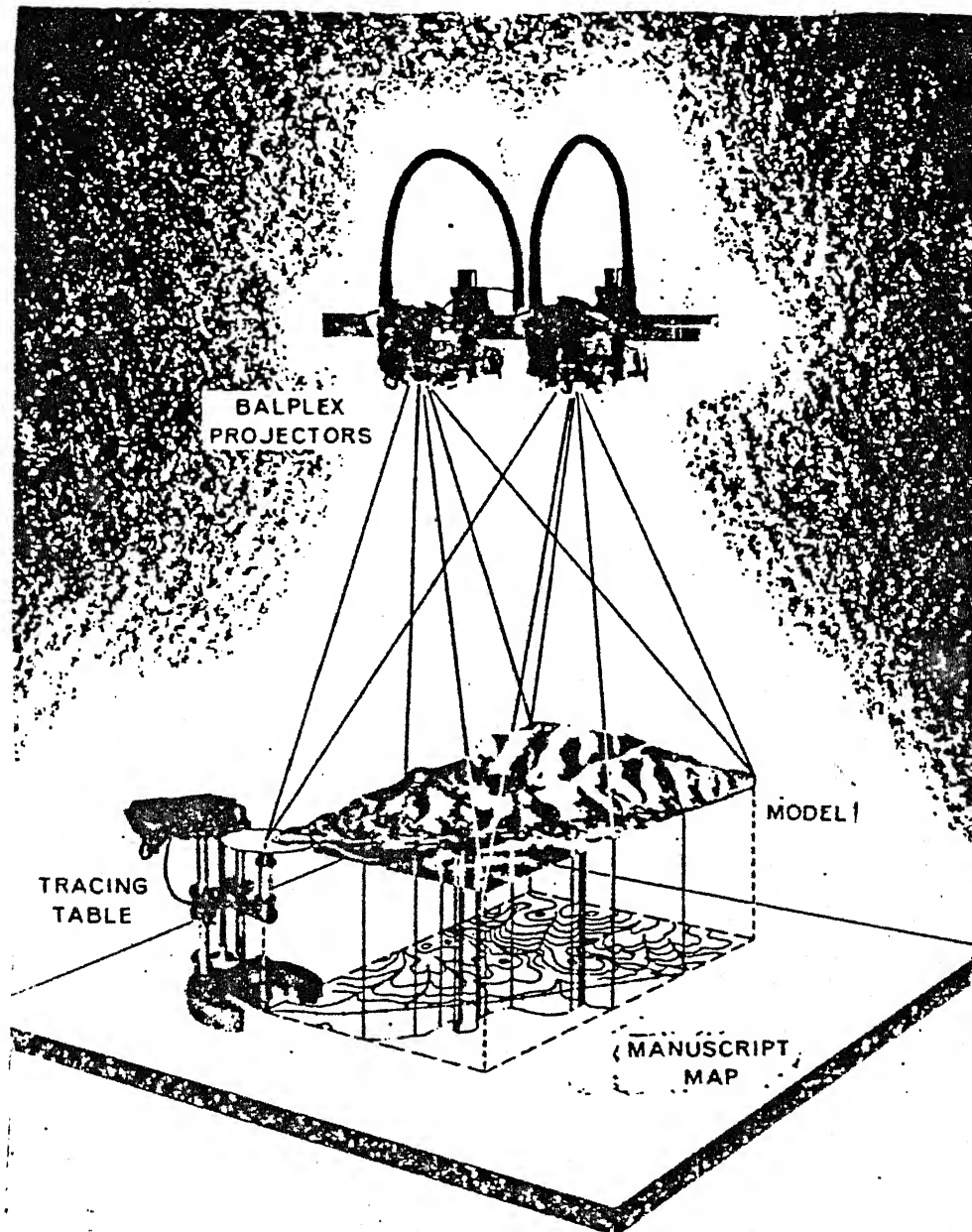


Fig 3.3 STEREOMODEL FORMATION.

[Ref 12 Page 266]

terrain an overlapping pair of diapositives or transparencies are placed into two projectors of photogrammetric machines and are reprojected. This procedure is called interior orientation and is shown in Fig.3.2.

In the process of reprojection the two adjoining projectors should be provided with such movements as to correspond with the possible camera positions at the time of photography so that by their mutual adjustments, the same relative position of adjoining photographs as during photography can be obtained. The process is called relative orientation and creates, in miniature a true three-dimensional stereo model of the overlap area.

After relative orientation, absolute orientation is performed. In this process the model is brought to desired scale and leveled with respect to reference datum. Fig.3.3 illustrates a stereo model created by interior, relative and absolute orientation of a stereo pair in the projectors or a Balplex plotter. For measurements, the position of any point is determined by bringing a reference mark into contact with model point. Planimetric positions (X,Y) of points are plotted by means of a pencil located vertically beneath the reference mark, and elevation (Z) are read directly from dial and which records up and down motion of platen, the dial having been indexed to ground control during absolute orientation. Fig.3.4 describes the principal components of a typical direct optical projection stereo plotter.

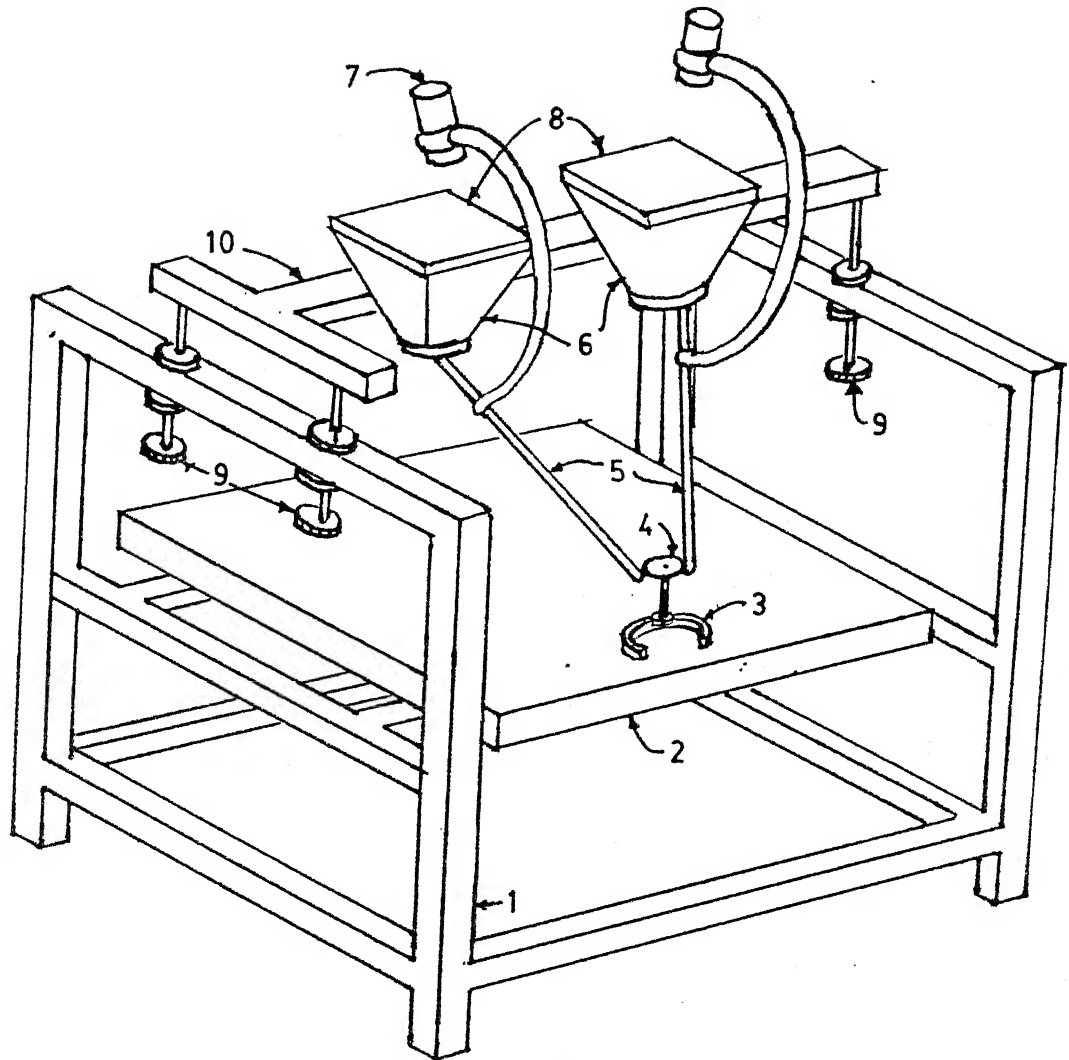


Fig 3.4 PRINCIPAL COMPONENTS OF STEREOPLOTTER.

## PRINCIPAL COMPONENTS OF STEREOPLOTTER

- (1) Main frame
- (2) Reference table
- (3) Tracing table
- (4) Platen
- (5) Guide rods
- (6) Projectors
- (7) Illumination lamps
- (8) Diapositives
- (9) Leveling screws
- (10) Projector bar

### 3.1.2(a) Data Collection from Stereo Plotters

With the analytical plotter two photographs are stereoscopically viewed with a special optical system (Fig.3.5). In the centre of the stereo photo there is a measuring mark. The coor-dinates of terrain point, where the measuring mark can be seen are stored in the computer as model coordinates. If the operator changes the coordinates, normally by turning hand wheels, the computer moves the photos in front of the optical system so that one can see the measuring mark at the point with the new coordinates. The task of the computer is, besides moving photos, putting in the parameters, doing the interior, relative and absolute orientation and error correction. The task of the operator is to produce input data for the different processing models. According to the mode, the operator positions the measuring mark on single point or follows lines. The registration of the coordinates will be done by the operator or automatically in fixed time or space intervals. The control of the servos and thus the motion of the photos is automatically carried out by computer.

Within each stereo model, an X-Y horizontal coordinate system is established with X-axis in the general direction of proposed highway alignment. Profiles in Y-direction are taken at regular intervals, and the Z-elevation along these profiles is recorded either at regular intervals of Y or at significant breaks in terrain slope, or both.

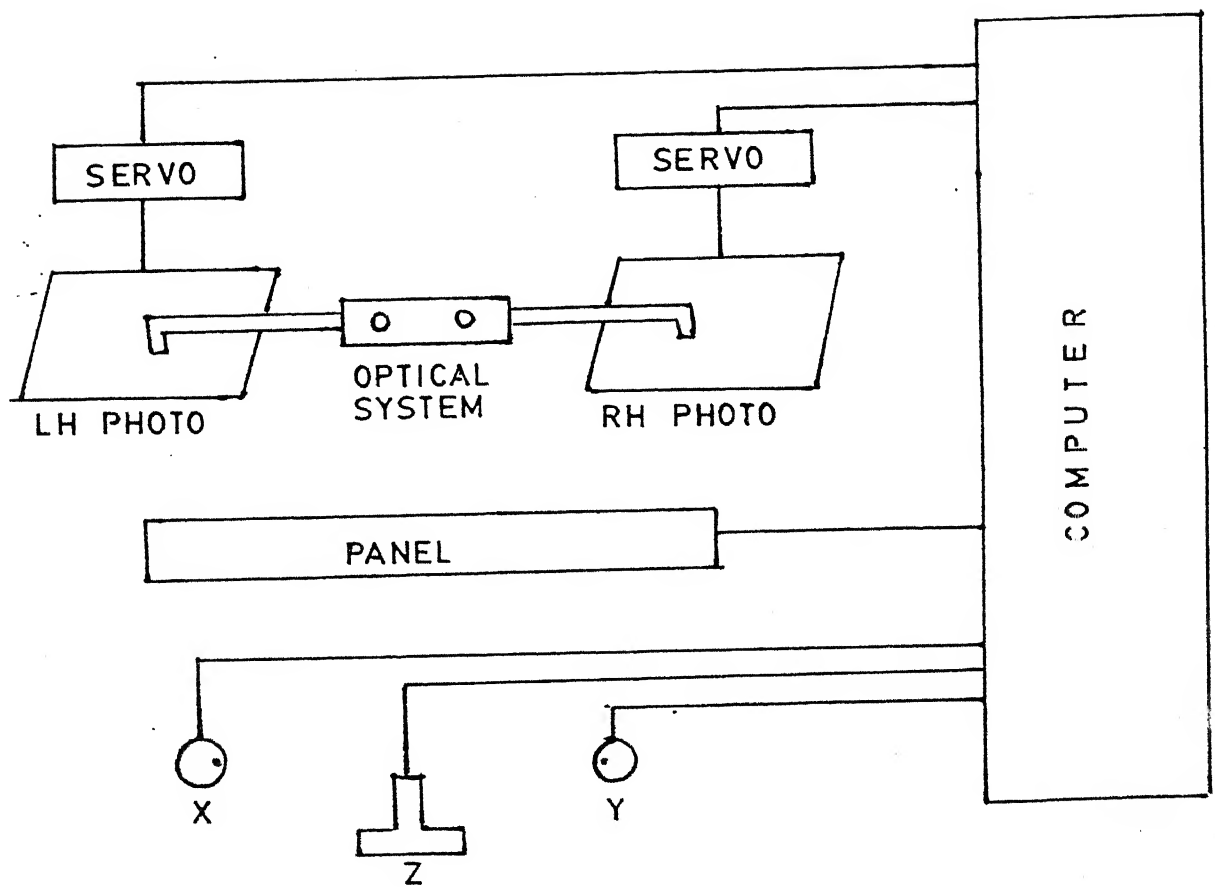


Fig 3.5 PRINCIPLE OF ANALYTICAL SYSTEMS.

### 3.1.3 Topographic Maps

A map is a two-dimensional representation of an area. Maps are important to photogrammetrists for they are compact and efficient means of expressing relationships and details. The two ways of data collection from a map are by digitizing contours (this will be discussed in sampling patterns) and from spot heights available from the map. The pattern of points on a map may be classified into (a) points regularly spaced on a grid or network (b) points scattered at random (c) points grouped in clusters. This is illustrated in Fig.3.6.

For the present study, the spot heights and contours from Map (restricted) are collected. The point distribution is scattered at random. The scale of the map is 1:10000. Spot heights are collected taking left bottom corner as origin. Heights are collected in meters. The matrix of data is used for the analysis of undulated terrain. For flat terrain analysis, the data is collected from reference [9], as the real world data was not readily accessible.

In general one or combinations of these sources is used. The problem of which points on the ground surface should be selected to represent the terrain can be split up into the choice of a pattern at point registration (sampling pattern) and an adequate sampling density.

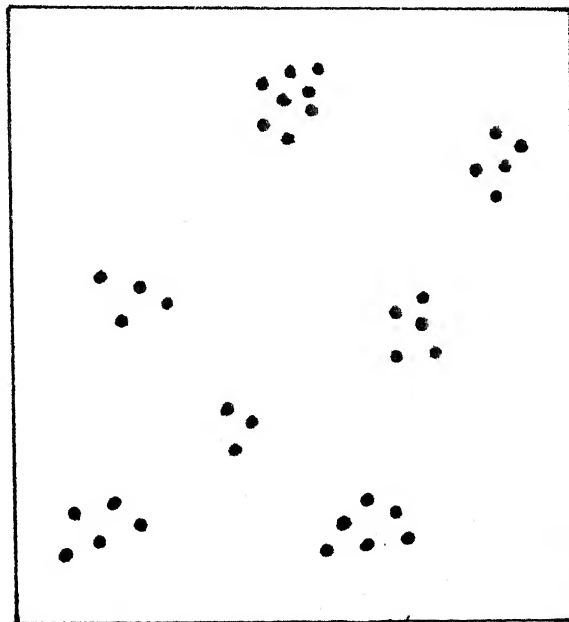
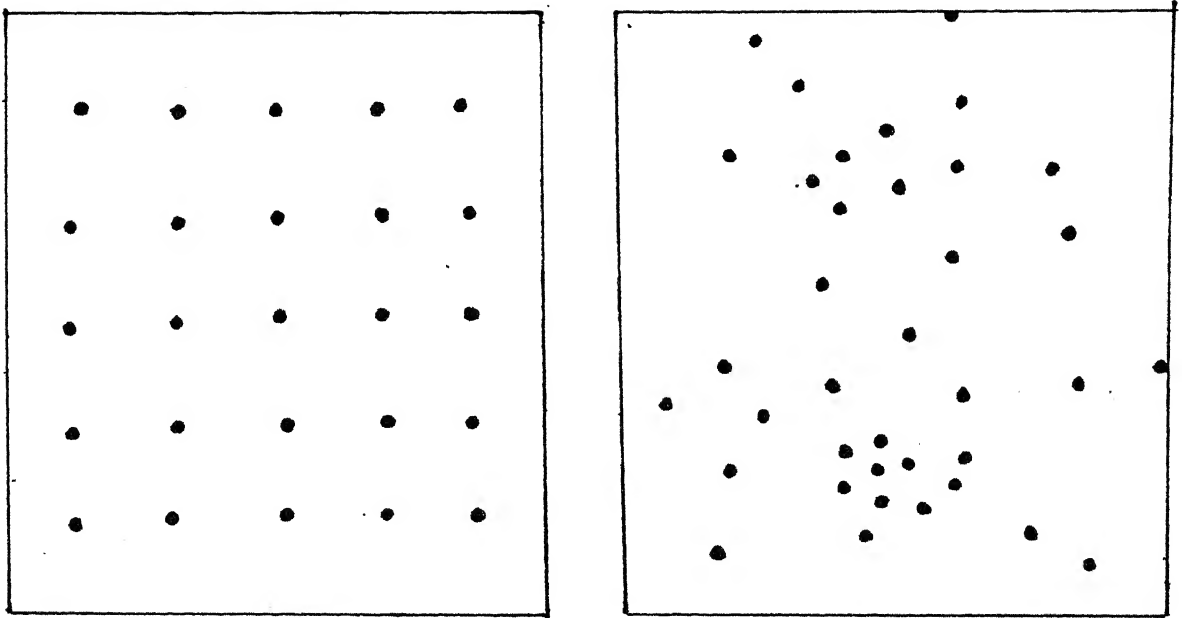


Fig 3.6 PATTERN OF POINTS ON MAPS.

### 3.2 Sampling Patterns

There are numerous possible sampling patterns, the most frequently used ones are discussed briefly here.

#### a) Homogeneous Grids

A rectangular or square grid is used to define the position of every point where a height has to be measured. The obtained Z coordinates are stored in matrix form.

Advantages : The photogrammetric model can effectively be used for the sampling process. The sampling does not require a great deal of expertise, and can be automated. A simple interpolation and retrieval system is possible. A DTM system based on a regular grid can be implemented on a relatively small computer.

Disadvantages : The cost-accuracy relationship is not likely to be good. (In uniformly shaped areas too many points will be measured and in strongly varying terrain too few, in general).

#### b) Contour Lines

Points are sampled along lines with a constant height, either in a photogrammetric model or on a topographic map. The obtained X and Y coordinates of the points are stored sequentially for each given height value.

Advantages : This pattern of registration is much more related to the terrain features than a homogeneous grid. Therefore they have the potential to achieve a good accuracy economy

relationship. Digitizing contours or plotting contours in a photogrammetric instrument does not require a high degree of expertise.

Disadvantages : Continuous sampling is less accurate than point by point sampling. The sampling process can hardly be atomized. Data compression procedures will be necessary, in general. Interpolation is awkward. A rather large computer is required to operate this system efficiently.

#### c) Morphologic lines and points

Significant terrain points (e.g., peaks, troughs) and terrain break lines (e.g., ridges, rivers, road edges) are measured, which gives a more or less random point pattern.

Advantages : A minimum number of points are measured  
Valuable in delineating man made features.

Disadvantages : The point selection would have to be carried out by a trained engineer or photogrammetrist.  
The sampling cannot be automated.  
Interpolation is elaborate.

#### d) Heterogeneous grids

B.Makarvic developed a method (progressive sampling) where a square is used as pattern of registration but the height is measured only at those intersection points where it is necessary. This very promising method is described in the next chapter.

### 3.3 Sampling density

It is possible to achieve any accuracy of the desired output of a DTM system(i.e. the reconstructed terrain) by varying the density of sampling, no matter what pattern of registration is used. Any increase in accuracy will, however, mean that the cost of acquisition will also increase. Normally for flat terrain the data points are collected at a distance, for undulated the data points are collected closely.

### 3.4 Data processing

The problem of deriving from the stored terrain data (e.g. contours or heights in a regular grid, or profiles) some special information (e.g. data for hill shading, volumetric data, contours, ...) is first of all one of interpolation.

A number of different methods are known and used in practice to determine the height for any intermediate point from the given X,Y,Z coordinates. They range from linear interpolation to quite sophisticated concepts like linear least squares interpolation. Detailed description of various interpolation methods are described in next chapter.

## CHAPTER IV

## MATHEMATICAL FORMULATION AND ALGORITHMS

4.1 Introduction

In this chapter the description of various algorithms of mathematical procedures that are used in this work are presented. They are

- (1) weighted average method
- (2) Interpolation methods
- (3) Progressive sampling method

The first step in DTM is to convert discrete data into gridded pattern. Size of a grid is usually taken as 20 mts. Normally in DTM photographs or maps of 1:10000 scale are used for data collection. So the size of a grid on a photograph or on a map will be 2mm which can be easily measured. If we take 10 mts grid more points are generated and storage becomes difficult.

4.2 Weighted average method Algorithm

(1) Gridding is the process of determining the values of the surface at a set of locations that are arranged in a regular pattern, usually a square from irregularly located control points where the values of the surface are known.

(2) As the first step control points must be sorted according to their geographic coordinates. Normally four control points are used to estimate elevation at grid node as four points are assumed to be in four directions.

(3) Fig 4.1(a) shows a series of observations on map. Each point is characterized by its X coordinate, Y coordinate and Z

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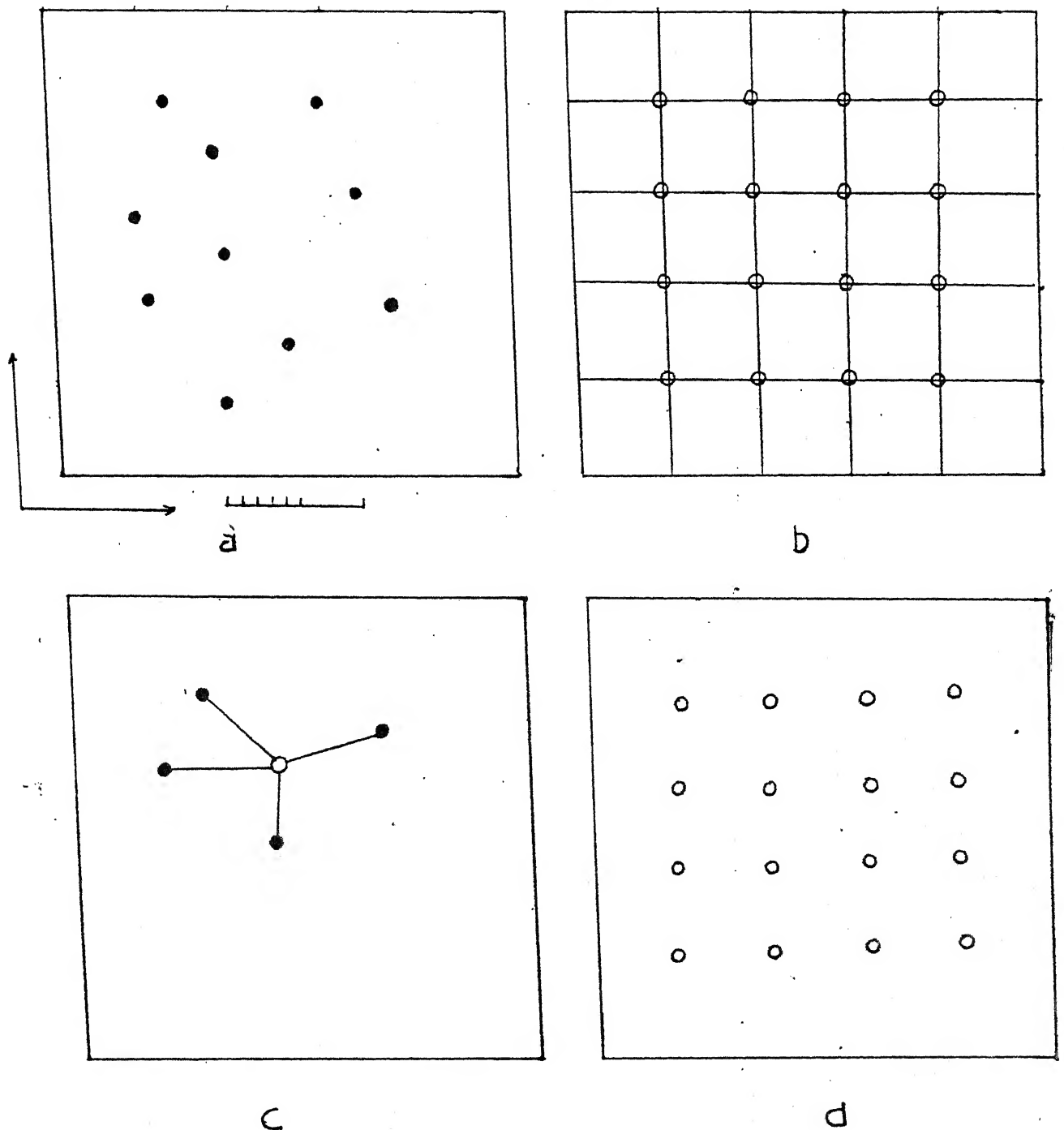


FIG 4.1

- (a) Original set of irregularly spaced points on a map.
- (b) Regularly spaced grid network, with nodes whose values are to be calculated.
- (c) Location of four nearest control points to one grid node.
- (d) Complete grid with estimated elevations at every node.

coordinate or elevation. The observations may be identified by numbering them sequentially as they are read in from 1 to I. Therefore an original data point I has coordinates  $X_i, Y_i$ , and  $Z_i$ .

(4) In Fig.4.1b, we have superimposed a regular grid of nodes on the map. These grid nodes are numbered sequentially, from 1 to k. Grid node k has coordinates  $X_k, Y_k$  and has an estimated elevation of  $Z_k$ .

(5) Now the next step is to search four nearest data points for each grid point as shown in Fig.4.1c.

(6) After searching four nearest data points to grid node k the distance  $D_{ik}$  from observation point i to grid node k is found by Pythagororous equation:

$$D_{ik} = [(X_k - X_i)^2 + (Y_k - Y_i)^2]^{1/2}$$

Having found the distances  $D_{ik}$  to four nearest data points we now can estimate the grid node elevation  $Z_k$  from these. The estimate is

$$Z_k = (Z_i/D_{ik}) / \sum_{i=1}^4 (1/D_{ik})$$

(7) In a similar manner we can evaluate the remaining grid nodes on the map. The computed grid with all values of  $Z_k$  posted is shown in Fig.4.1d.

#### 4.3 Interpolation Procedure

Using the method of weighted average method the elevations at four corners of each grid can be calculated. In progressive sampling to locate new point we need a grid with nine points called "Zero run" and from X,Y coordinates of the new point we should calculate Z value. To solve this problem the best way is to express Z as a function of X,Y so that structure required for Zero run as well as for further interpolation Z values can be

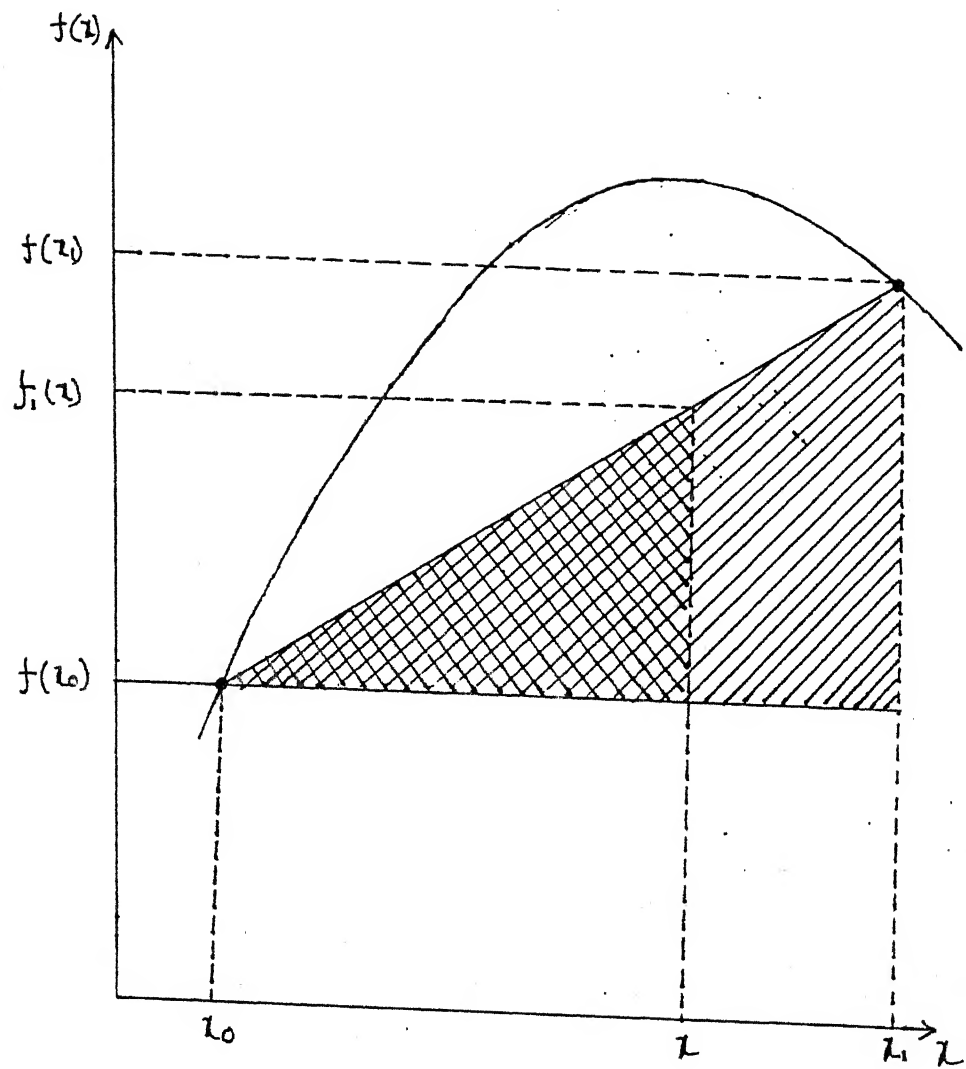


Fig.4.2 GRAPHICAL DEPICTION OF  
LINEAR INTERPOLATION

There are a number of different interpolation procedures to determine the height for any intermediate point from the given X,Y,Z coordinates. They are

- (1) Linear Interpolation
- (2) Least Square Interpolation

#### 4.3.1(a) Linear Interpolation

The simplest form of interpolation is to connect two data points with a straight line. The technique is called linear interpolation and is depicted graphically in Fig.4.2.

Using similar triangles

$$[f_1(X) - f(X_0)]/(X - X_0) = [f(X_1) - f(X_0)]/(X_1 - X_0)$$

which can be rearranged to yield

$$f_1(X) = f(X_0) + \{[f(X_1) - f(X_0)]/(X_1 - X_0)\} (X - X_0)$$

In general, the smaller the interval between the data points, the better the approximation.

#### 4.3.1(b) Bilinear Interpolation

Here it is assumed that Z varies linearly in both X and Y directions.

A bilinear polynomial  $Z = A + BX + CY + DXY$  would suit a undulated terrain. This is illustrated in Fig.4.3.

The bilinear constants are

$$A = Z_1 \quad B = (Z_2 - Z_1)/\Delta X$$

$$C = (Z_3 - Z_1)/\Delta Y$$

$$\text{and } D = (Z_1 + Z_4 - Z_2 - Z_3)/(\Delta X * \Delta Y)$$

Bilinear constants are calculated and they are used for further interpolation in progressive sampling and Earthwork calculation.

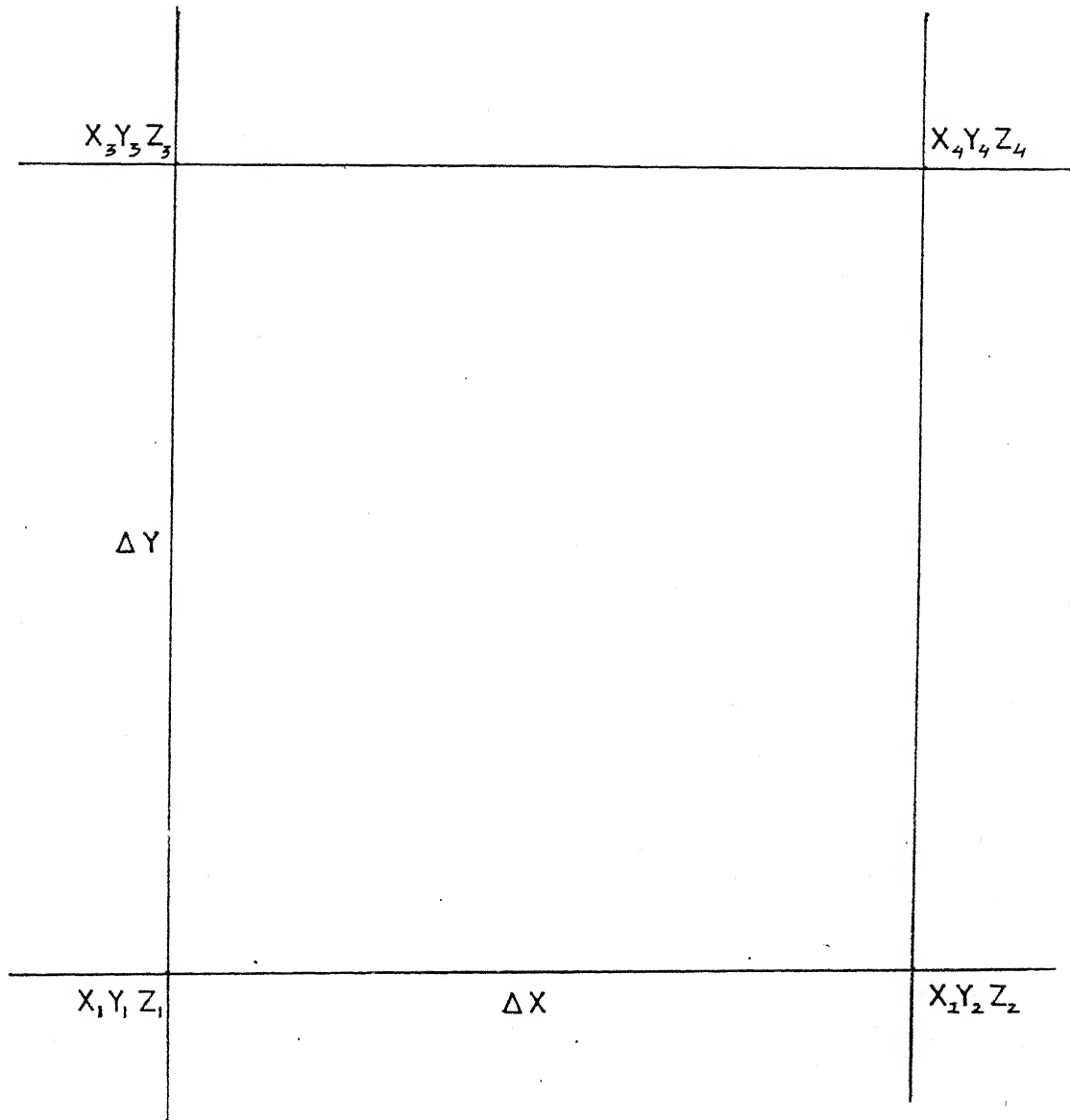


Fig.4.3 GRAPHICAL DEPICTION OF  
BILINEAR INTERPOLATION

#### 4.3.2. Least square Interpolation

This method was developed by Legendre over 150 years ago and this has been used in Statistics for many years. Earth, assumed to be a plane surface was represented by mathematical model

$$Z = f(X,Y) = L + MX + NY$$

L,M,N are the parameters and (X,Y,Z) represent any point on the plane surface.

Graphical depiction of multiple linear regression where Z is a linear function of X and Y is shown in Fig.4.4.

The best values of coefficients are determined by setting up the sum of the squares of residuals

$$S_r = \sum (Z_i - L - MX_i - NY_i)^2$$

and differentiating with respect to each coefficient

$$\begin{aligned} \delta S_r / \delta L &= -2 \sum (Z_i - L - MX_i - NY_i) \\ \delta S_r / \delta M &= -2 \sum X_i (Z_i - L - MX_i - NY_i) \\ \delta S_r / \delta N &= -2 \sum Y_i (Z_i - L - MX_i - NY_i) \end{aligned} \quad (4.1)$$

The coefficients yielding the minimum sum of the squares of the residuals are obtained by setting the partial derivatives equal to zero and expressing Eq.(4.1) as a set of simultaneous linear equations.

$$\begin{aligned} nL + M \sum X_i + N \sum Y_i &= \sum Z_i \\ L \sum X_i + M \sum X_i^2 + N \sum X_i Y_i &= \sum X_i Z_i \\ L \sum Y_i + M \sum X_i Y_i + N \sum Y_i^2 &= \sum Y_i Z_i \end{aligned}$$

For flat terrain where height variation is less than 10%, this model can be adopted. These constants are used for further interpolation.

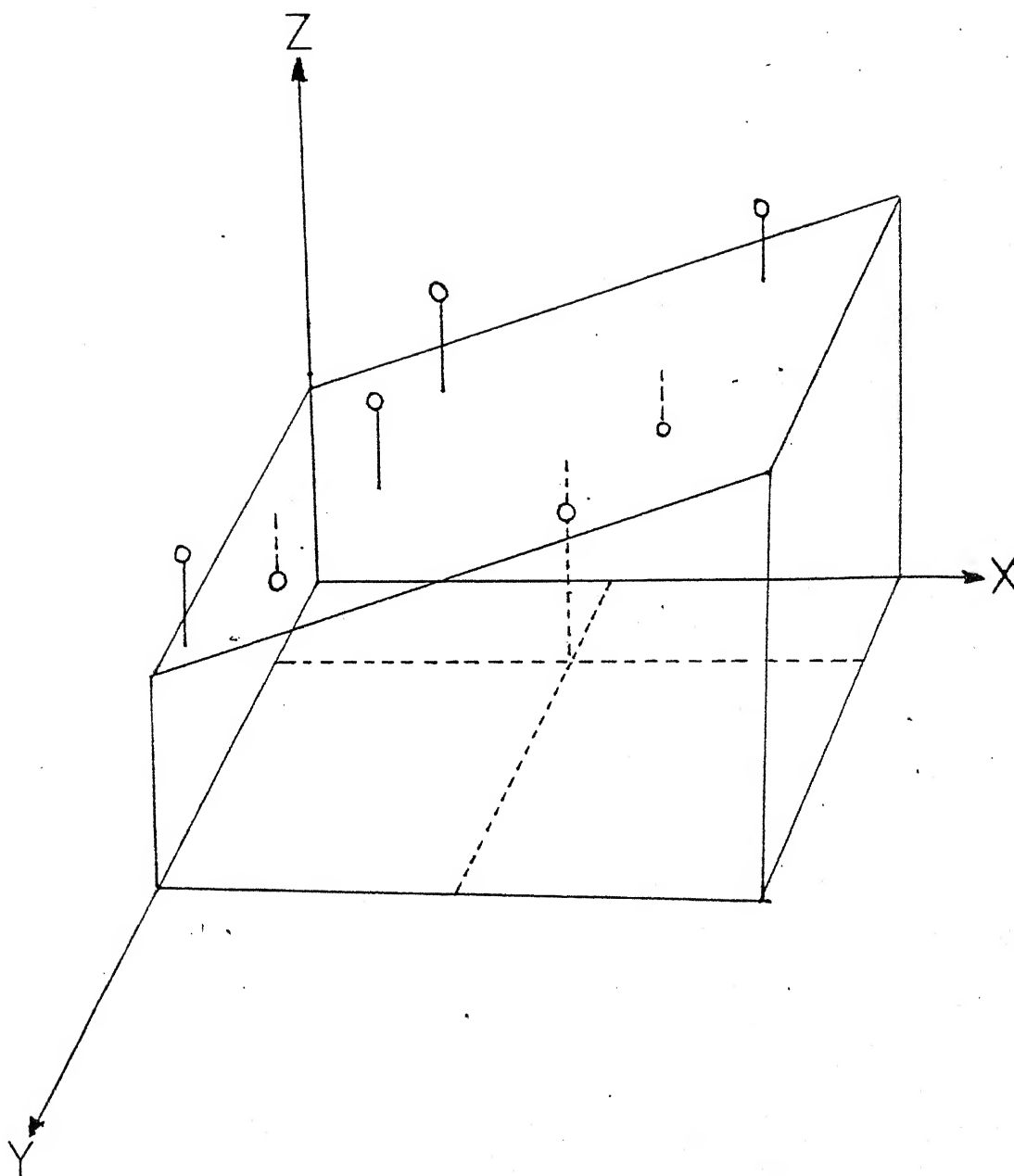


Fig.4.4 GRAPHICAL DEPICTION OF  
MULTIPLE LINEAR REGRESSION

#### 4.3.2.(b) Collocation

The method of collocation, sometimes also known as advanced least squares interpolation has been extensively applied to solution problems related to geodesy and photogrammetry. The back bone of the method is so called covariance function, which takes into account the effect of correlation between observations.

The basic assumption made here is that for a pair of points on the surface of earth, the pair of heights are correlated according to the distance between points. This means that the distance and direction are involved in the function which is ultimately to be used for derivation of height differences. Thus, if we have a set of given points with heights  $h_1, h_2, h_3, \dots, h_n$ , then height 'h' of an unknown point can be written in function form as

$$h = f(r_1, r_2, \dots, r_n, \theta_1, \theta_2, \dots, \theta_n)$$

where  $r_1, r_2, \dots, r_n$  and  $\theta_1, \theta_2, \dots, \theta_n$  are the distances and directions of the given points from unknown points (height only).

This method is elaborately described in references quoted under [14, 16, 17]

### 4.3.3 Statistical Analysis

Statistical analysis is carried out to express the accuracy of the model.

As a first step variance covariance matrix for regression coefficients is computed.

Variance is computed by dividing corrected sum of squares by degree of freedom. Variance is defined as the average square deviation from mean.

covariance is the joint variation of two variable about their common mean. This measures the distribution of values around common mean.

Covariance is calculated by dividing corrected sum of products by degree of freedom.

Secondly correlation matrix for regression coefficients is computed. To estimate the degree of interrelation between variables correlation coefficient 'r' is calculated. Correlation is the ratio of covariance of two variables to the product of their standard deviations.

Posterior variance ( $\sigma^2_o$ ) is calculated to express absolute accuracy.

$$\sigma^2_o = [ (\text{Sum of squares of residuals}) / (n-1) ]^{1/2}$$

### 4.4 Progressive Sampling

Progressive sampling of discrete points arranged in a regular grid of varying density provides a powerful tool for the acquisition of digital terrain data. The sampling process is started by measuring a low density point grid. The corresponding

data are analyzed to locate the positions of new points to be sampled in the next run.

The algorithm and logic of the method are explained below.

#### Algorithm

- (1) For organizational reasons and to reduce the required computer storage the entire area is divided into grids.
- (2) The heights at four corners are known to us from the output of weighted average method. The heights at nine points are calculated.

This structure is called zeroth run and is shown in fig4.5.

- (3) Progressive sampling is supposed to be carried successively for all grids in an ordered manner. For transition from one grid to next, the boundary data should be kept temporarily in the computer storage.

- (4) In numerical processing the first step of the analysis is computation of height differences between the adjacent points along each row and column

$$Z_{i,j-8} = Z_{i,j-16} - Z_{i,j}$$

$$Z_{i-8,j} = Z_{i-16,j} - Z_{i,j}$$

- (5) The next step is to calculate second differences separately for the rows and columns. This can be formulated algebraically for the point in the following way

$$Z_{1.25}(k,i-1) - Z_{1.9}(k,i) = \delta Z_{1.1}(r) \text{ for a row}$$

$$Z_{25.1}(k-1,i) - Z_{9.1}(k,i) = \delta Z_{1.1}(c) \text{ for a column}$$

The heights sampled in the following runs are processed similarly. When disregarding random terrain fluctuations, the second differences carry information about terrain curvature.

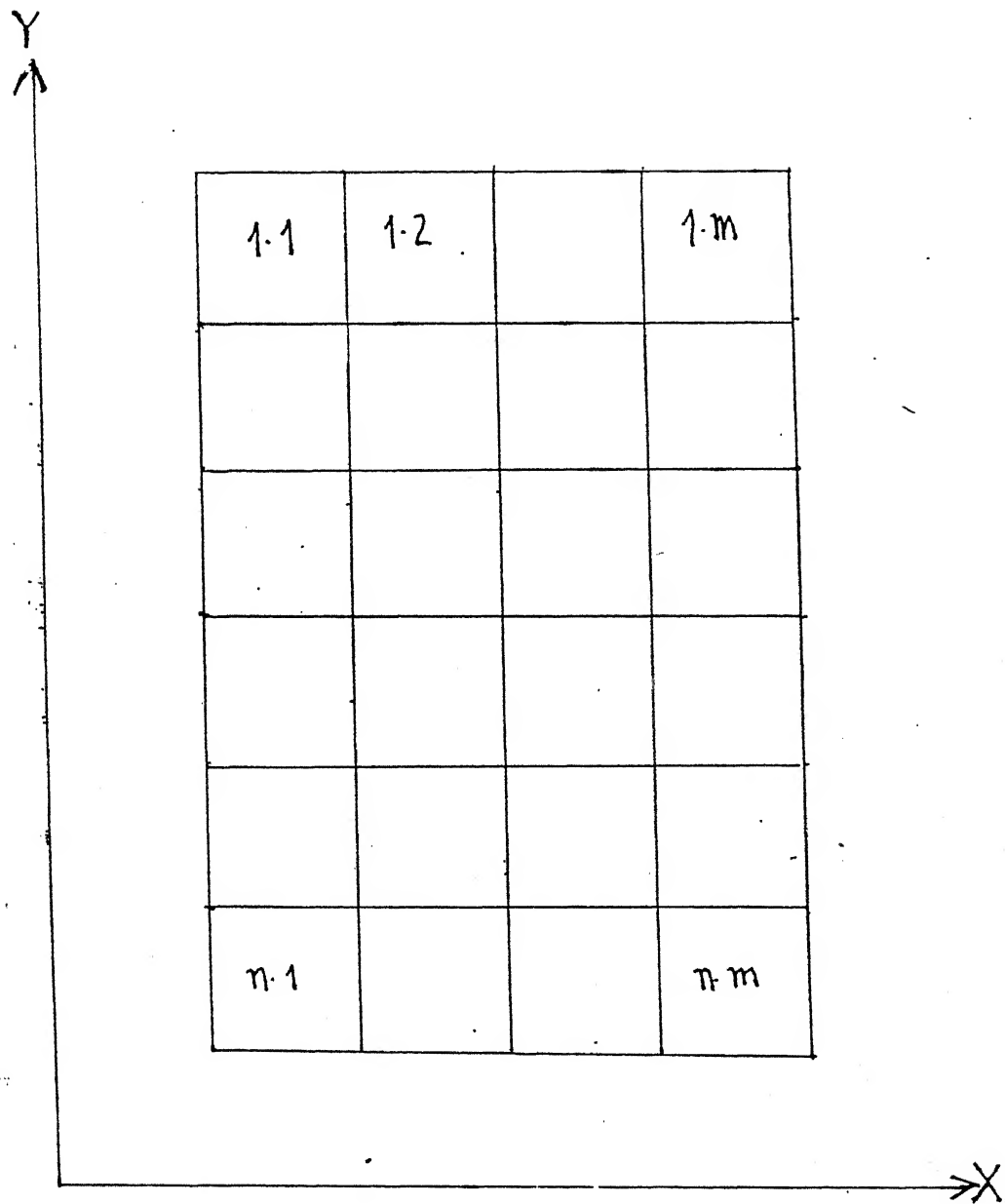


Fig.4.5 MODEL AREA SUB-DIVIDED  
INTO  $n \times m$  GRIDS

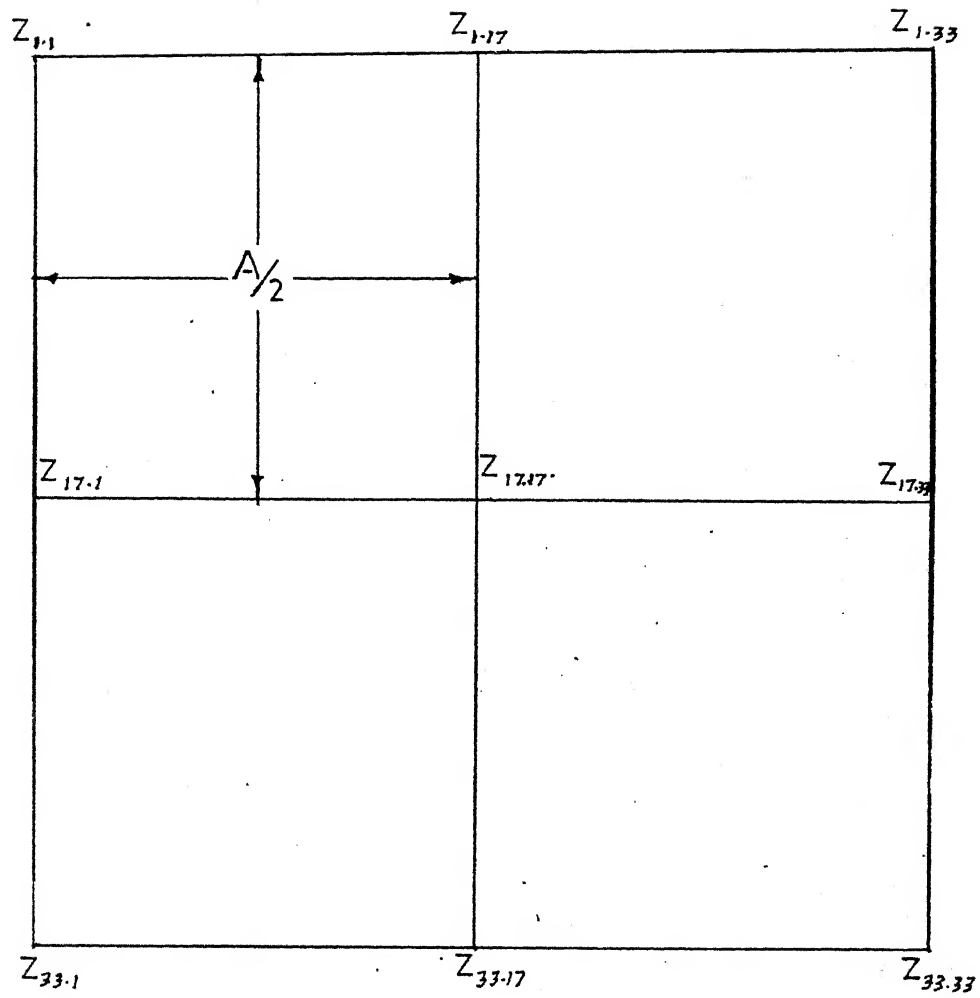
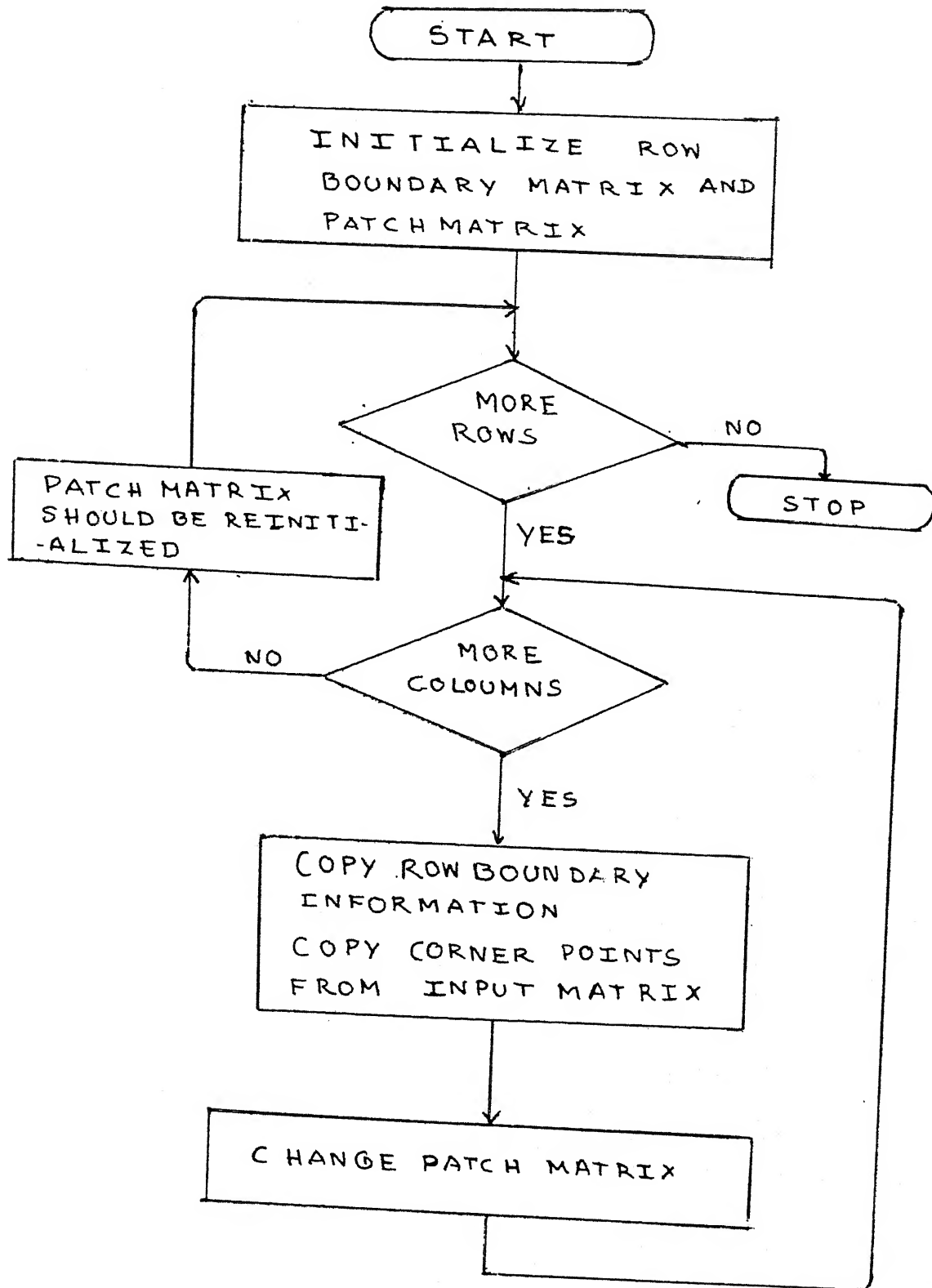


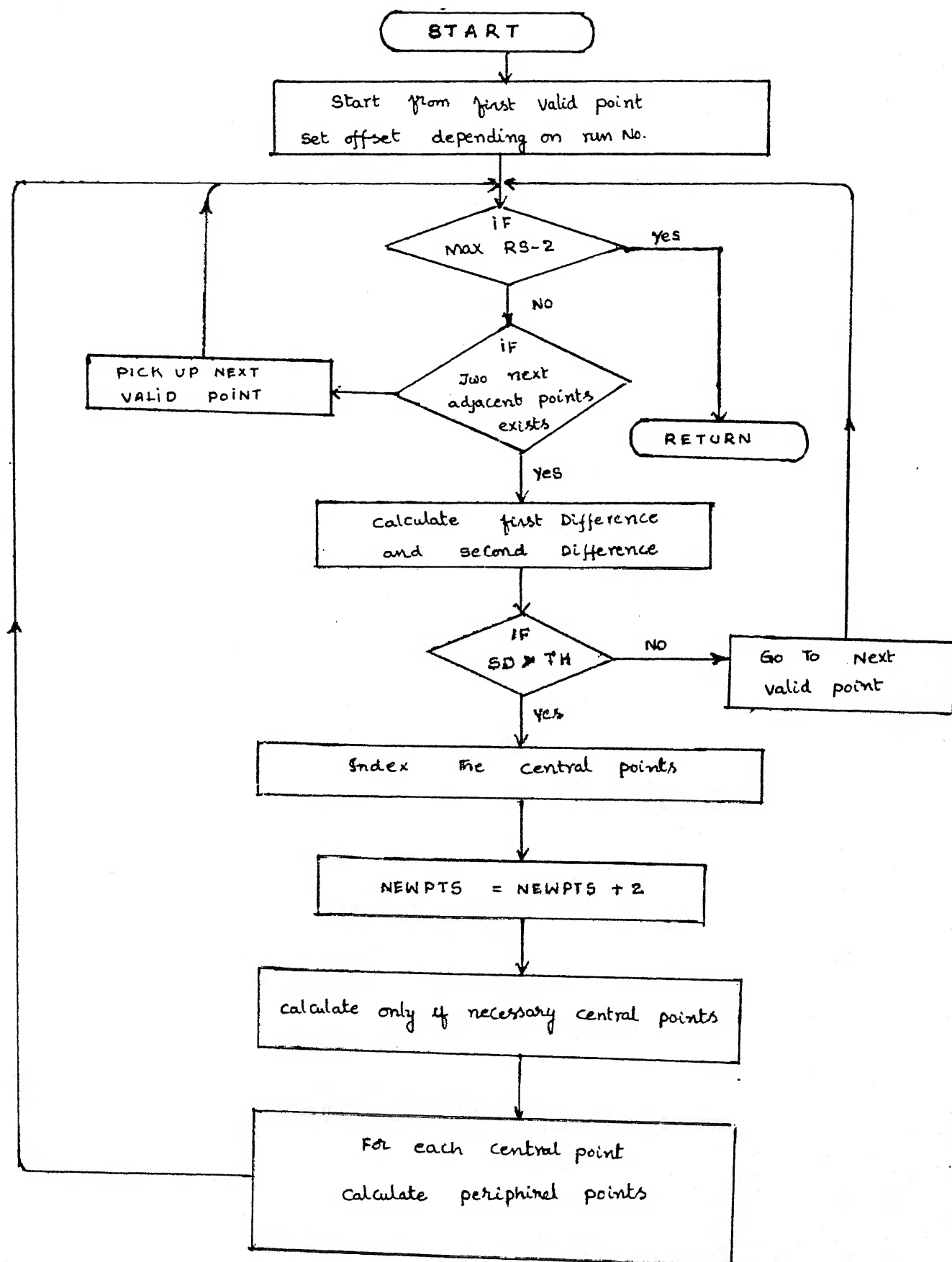
Fig.4.6, ZERO SAMPLE

When curvature is substantial, it is desirable to increase the sampling density.

- (6) For the selection of new points the following criteria may be applied. If the  $\delta Z_{i,j} \leq \delta th$  no further sampling is needed. If the second difference  $th$  exceeds a certain threshold the sampling density should be increased. The values of threshold for various types of terrains are taken from reference (4).
- (7) After the central points to be sampled are determined, the surrounding peripheral points should be considered for the next run. Steps (1) to (6) are to be repeated for all runs.
- (8) The low density grid with  $3 \times 3$  points is considered first and is densified upto a maximum of  $33 \times 33$  in successive runs.
- (9) The same procedure is repeated for all other grids.
- (10) The flow chart of computer operations involved is given in next page.



FLOWCHART OF PROGRESSIVE SAMPLING



Routine to process a row or a column in a patch in a run.

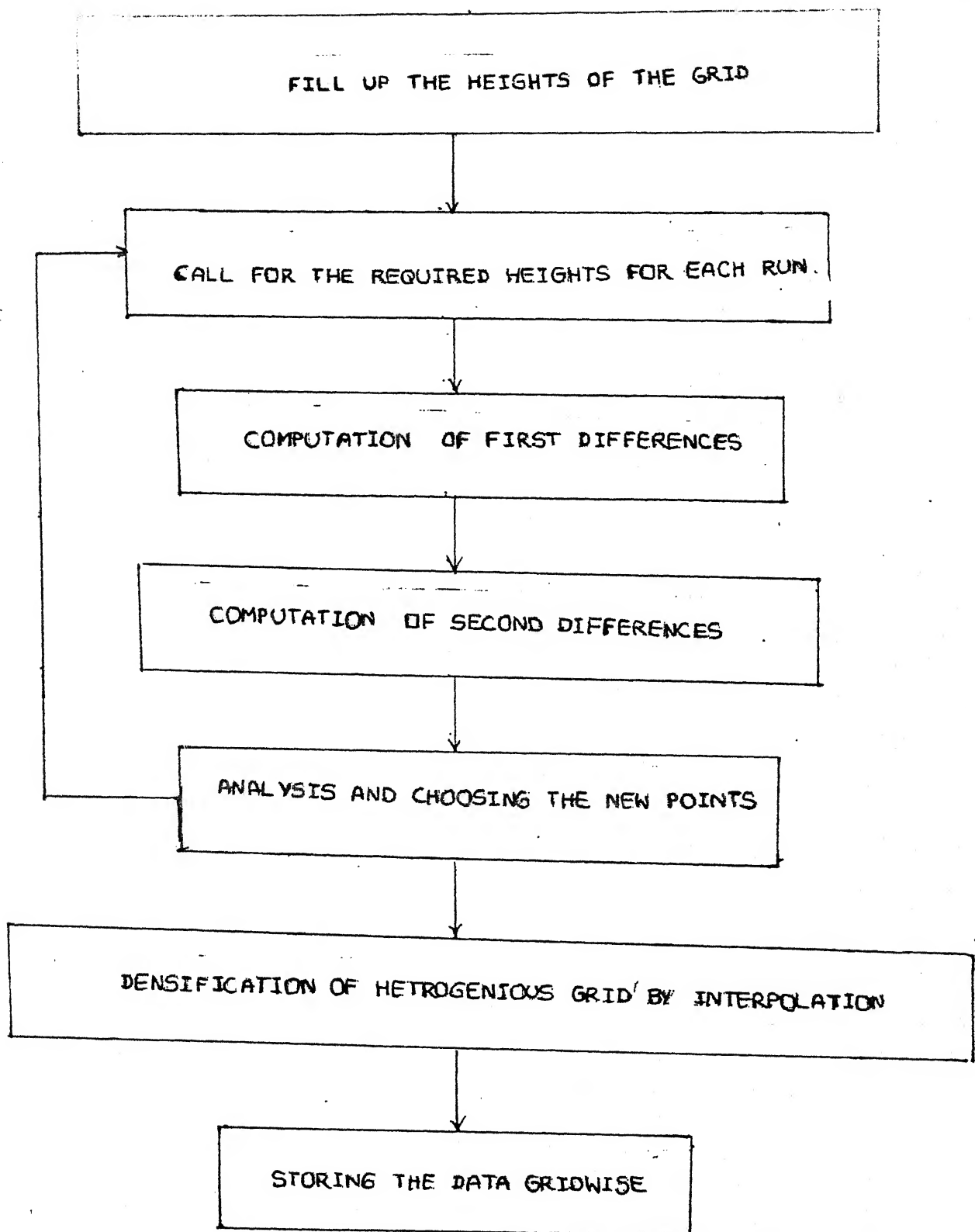


FIG 4.7. FLOW CHART OF COMPUTER OPERATIONS.

## CHAPTER 5

### VOLUMETRIC COMPUTATION

#### 5.1 INTRODUCTION

In this chapter the application of DTM to volumetric computation is emphasized. The methods mentioned in chapter two are elaborated here.

The ultimate purpose of field surveys is to obtain certain numerical values to represent quantities such as areas and volumes. Surveyors are often called upon to measure volumes for engineering projects viz.. highway design, water resources structures construction and building construction. Volumetric determination is an important aspect in practical surveying. Earth work operations involve the determination of the volumes of materials which must be excavated or embanked on engineering projects to bring the ground surface to predetermined grade. The field work involves the measurements of the dimensions of various geometrical solids which make up volumes, the setting of grade stakes, and the keeping of field notes. The office work involves the computations of the measured volumes and determination of most economical measures of performing work. Because of the inherent accuracy of modern topographic maps of large scale produced by photogrammetric methods much of the field work is reduced.

Volumetric determination by point grids of digital terrain model data offers the advantage of great flexibility in route location. The main reason for this is it has become possible to collect the point data from photographs in automatic modes of equal spacing using analytical and semi automatic photogrammetric instruments.

### 5.2 Volumetric computation using DTM :

The basic task of a highway engineer is to select a proper alignment between given two termini. For that he has to study (i) road alignment search procedure (ii) sight distance computation and (iii) earthwork computation. Then optimal route (s) will be decided. Surveyor plays an important role in earth work calculations.

Volumetric determinations are essential in highway design phase. More commonly, actual ground measurements are obtained for such determinations. Application of modern techniques of photogrammetry to highway design in developing countries is still more or less in an initial stage due to lack of knowhow. Research on this field is therefore limited. Use of photographs provides us additional information about types of soils and geological characteristics of terrain under observation.

The terrain surface is essentially defined by complete numerical study in X,Y, and Z coordinates of very large number of points distributed over the strip selected for the projects.

In developed countries, numerical photogrammetry and automatic computations are being more and more adopted for alignment and designing motorways. The X,Y,Z values taken as input to computer with a suitable mathematical processing model to obtain the volumes of cut and fill.

For the present study cross section area method is used for volume computation. Different cross sections are shown in fig 5.1. The cross section shown fig 5.1 (a) is adopted for volume calculation for flat terrain. For undulated terrain the cross sections shown in fig 5.1(b),5.1(c) can be used for volume calculation.

The logic of the computer program is shown in the flow chart. The results are presented in chapter 6.

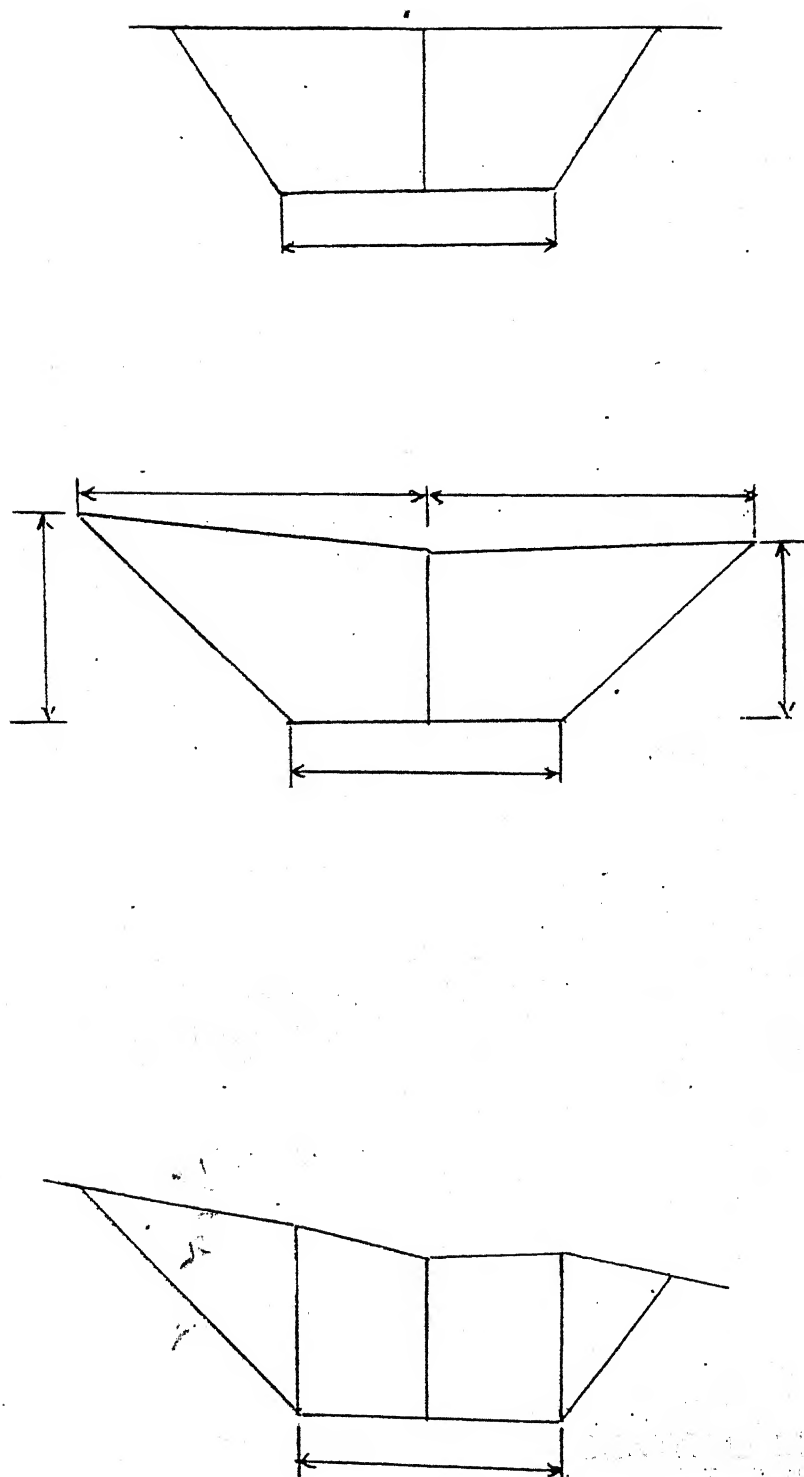
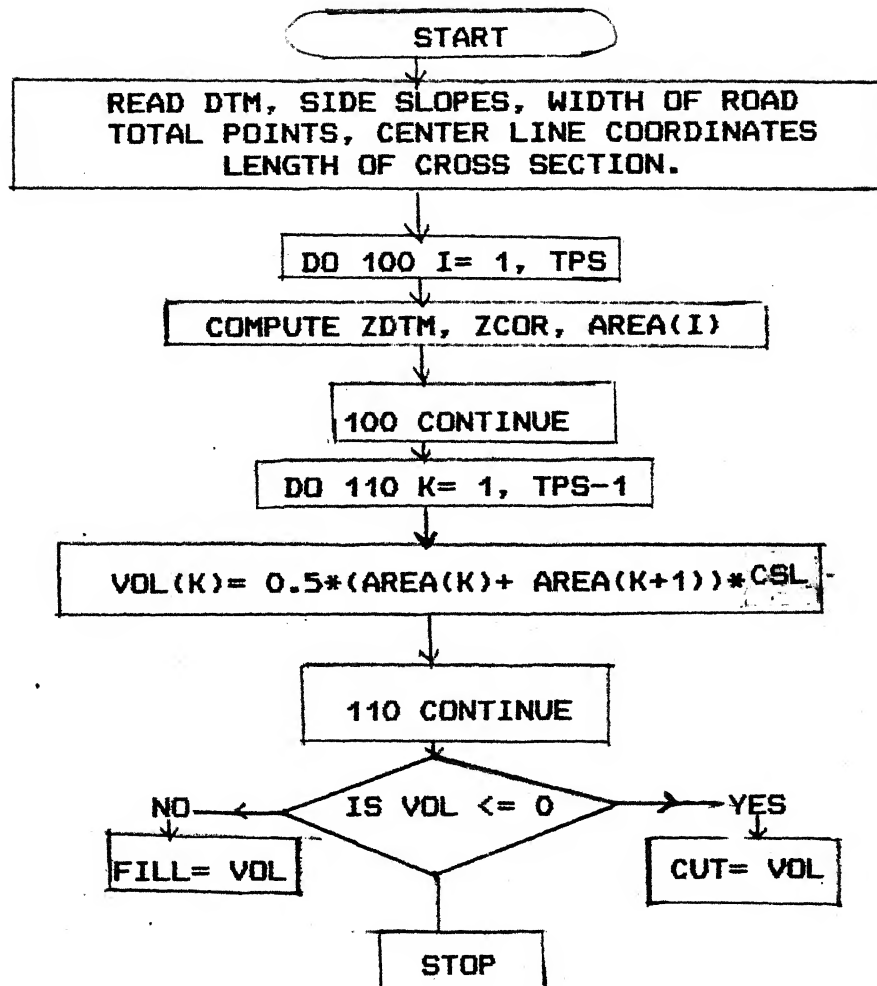


Fig. 5.1. EARTHWORK CROSS-SECTIONS



FLOW CHART FOR VOLUME COMPUTATION

## CHAPTER VI

## RESULTS

## 6.1 Analysis

In this chapter results are presented. Fig 6.1 shows the locations of data points collected for flat terrain. The x,y and z values of the data points are given in Table 6.1.1. The locations of points to form the model are shown in fig 6.2. Totally twenty points are taken to solve the model  $Z = L + MX + NY$ , so that there exists a degree of freedom of 17. The constants L, M and N are computed using STAT package. Posterior variance is computed to indicate absolute accuracy. The results are shown in Table 6.1.2.

Fig 6.3 shows the locations of data points collected for undulated terrain. The X, Y and Z values of the data points are given in Table 6.1.3. To analyse undulated terrain entire area is divided into five parts, as shown in fig 6.4. Five sets of each, Variance- Covariance matrices, correlation matrices for regression coefficients and posterior variances are computed. They are shown in Table 6.1.4 to Table 6.1.8.

## 6.2 Simulated terrain generation:

An oblique view of a surface provides a visually effective representation of slopes and peaks. Displaying a space curve or surface is not an easy task. The simplest method of displaying curves and surfaces is wire-frame technique. To display a curve, its coordinates are evaluated for many closely spaced points and these points are connected by short straight line segments. A complete surface is displayed by generating a mesh of such curves holding one parameter constant at a time.

Geometric transformations play an important role in generating images of three dimensional scenes. The important geometric transformations are translation, rotation and scaling. Then perspective transformations are carried out to display the three dimensional surface.

In the present investigation three dimensional terrain is simulated using PLOTIT software on IBM PC-AT with color monitor. The sequence of computer operations involved are shown in flow chart.

START

ENTER PLOTIT

INPUT DISCRETE X,Y,Z VALUES, NO. OF COLUMNS  
AND NO. OF ROWS.

INPUT DESIGN PARAMETERS, POLYNOMIAL GOVERNING  
THE SURFACE, VIEW ANGLE, BASE HEIGHT,  
PERSPECTIVE FACTOR.

INTERPOLATION

TRANSFORMATIONS

DISPLAY THE SURFACE

STOP

FLOW CHART FOR COMPUTER OPERATIONS IN  
SURFACE MODELING.

### 6.3 Volumetric Computation

Volumes are computed by using end area method. Three different routes between two given control points passing through different obligatory points are considered for volume calculation.

The parameters for volume calculation are digital terrain model, side slopes, width of the road and length of cross section. .

For the present study flat terrain is considered. The other design parameters are

Side slopes =1:1.5

Width of road =12 mts

Length of cross section = 40 mts

Tables 6.1.9 to 6.1.11 shows the results. Route I is the optimal route between two given control points with minimum earth work.

### 6.3 (a) Mass diagram:

Mass diagram or mass curve is a curve plotted on a distance base, the ordinate at any point of which represents the algebraic sum up to that point of the volumes of cuttings and fills from start of the earthwork or from any arbitrary point. In obtaining algebraic sum the convention is to consider fills negative and cuts positive. The positive total volumes are plotted above and the negative total volumes are plotted below the base line.

Uses of mass curve:

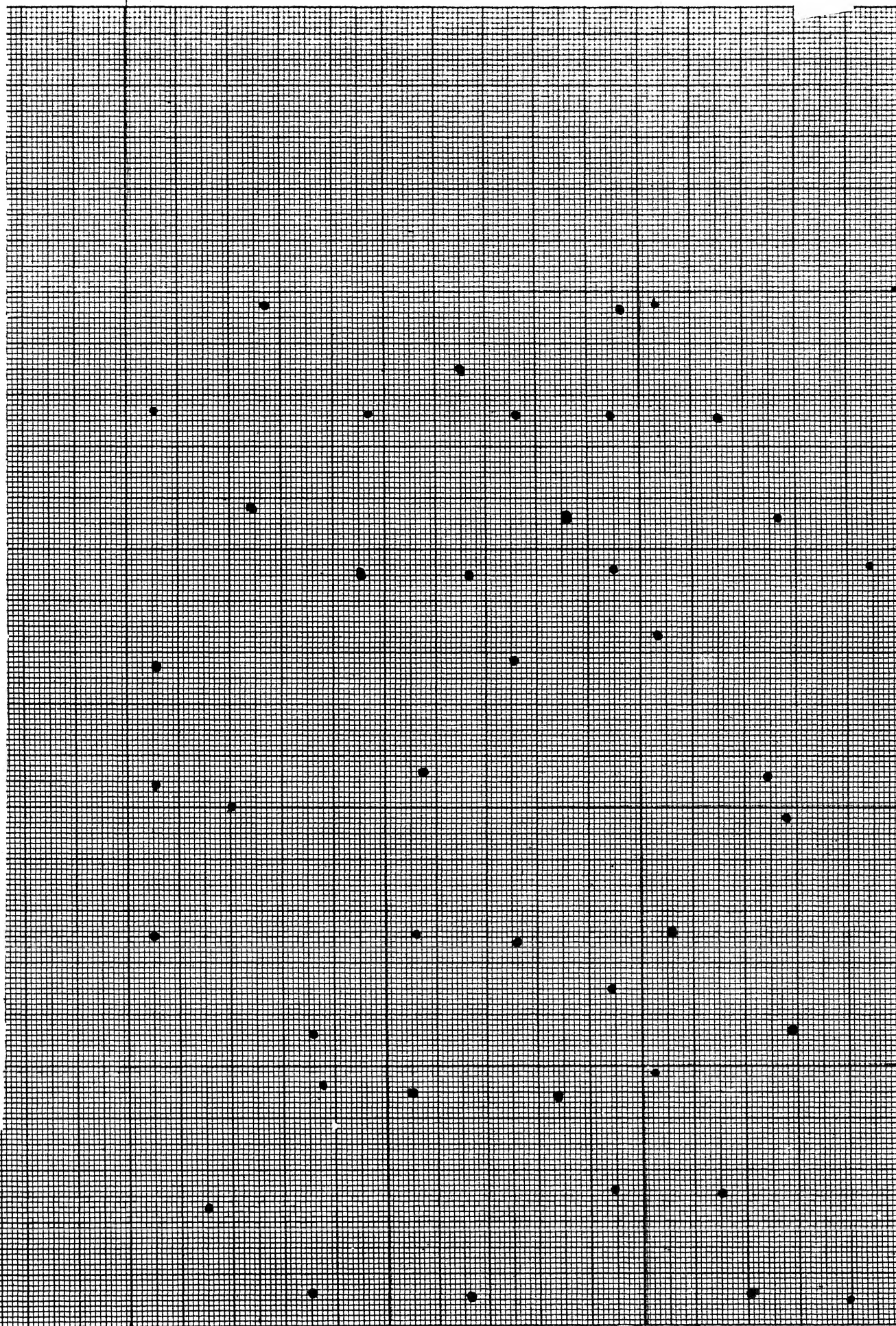
With the sign convention adopted upward slope of the curve in the direction of algebraic summation indicates excavation as downward slope indicates embankment.

Between any two points in which the curve cuts the base line the volume of excavation equals that of embankment, since the algebraic sum of the quantities between such points is zero.

The vertical distance between a maximum point and the next forward minimum point represents the whole volume of an embankment, that between a minimum and next forward maximum point the whole volume is cutting.

Fig 6.1 LOCATIONS OF DATA POINTS FOR FLAT TERRAIN

SCALE 1 cm = 20 mts.



## INPUT DATA FOR FLAT TERRAIN

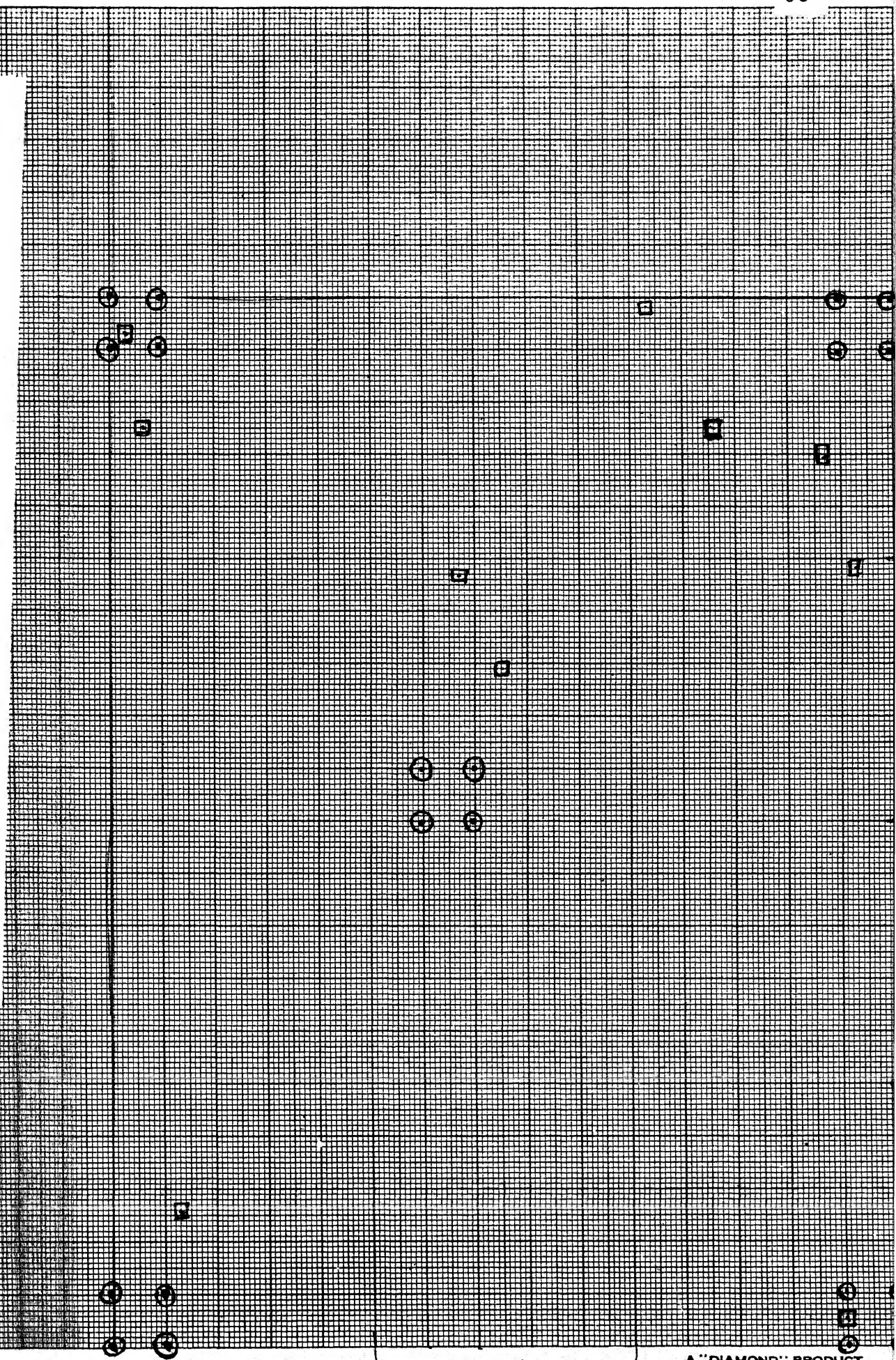
\*\*\*\*\*

TABLE 6.1.1

\*\*\*\*\*

10	20	204.5
40	70	204.9
90	85	205.1
115	42.5	204.2
195	43.00	204.1
129	88.5	204.1
211	50.0	204.1
266	93	204.6
295	12.0	205.3
312	45.00	204.7
350	68.00	204.1
398	95.0	204.4
12	168	204.5
52	112	204.3
88	132	205.0
90	190	205.1
130	110	205.2
148	148	204.4
150	188	204.3
216	186	205.3
258	150	205.4
288	168	204.0
291.5	110	205.0
312.5	127.5	204.5
352	111.5	204.8
396	108	205.0
370	170	205.2
12.5	230	205.7
43.5	270	205.0
92	224	205.1
111	228	204.0
150	290	204.1
200	260	204.9
208	290	204.5
253	288	204.3
290	210	205.3
315	250	204.0
353	207	204.2
354	288	205.0
398	294	204.2

Fig 6.2 ○ LOCATIONS OF DATA POINTS TO FORM MODEL  
□ LOCATIONS OF POINTS TO TEST MODEL  
SCALE 1 cm = 20mts.



**6.1.1a Analysis of flat terrain:**

The Variance- Covariance matrix for Regression Coefficients:

Coeff.	L	M	N
L	0.070	-1.30E-04	-1.77E-04
M	-1.30E-04	6.50E-07	0.0
N	-1.77E-04	0.0	1.15E-06

Correlation matrix for Regression Coefficients:

Coeff.	L	M	N
L	1.00	-0.607	-0.620
M	-0.607	1.00	0.0
N	-6.20	0.0	1.0

Regression Coefficients:

$$L = 204.551$$

$$M = -8.33E-05$$

$$N = -0.001$$

Table 6.1.2: MODEL  $Z = L + MX + NY$ 

Z observed	Z fitted
204.50	204.53
205.30	204.49
204.20	204.51
205.40	204.22
204.10	204.37
205.30	204.45
205.30	204.24
205.30	204.23
204.40	204.42
204.00	204.35

Posterior variance = 0.7308729

Table 6.1.2(a)

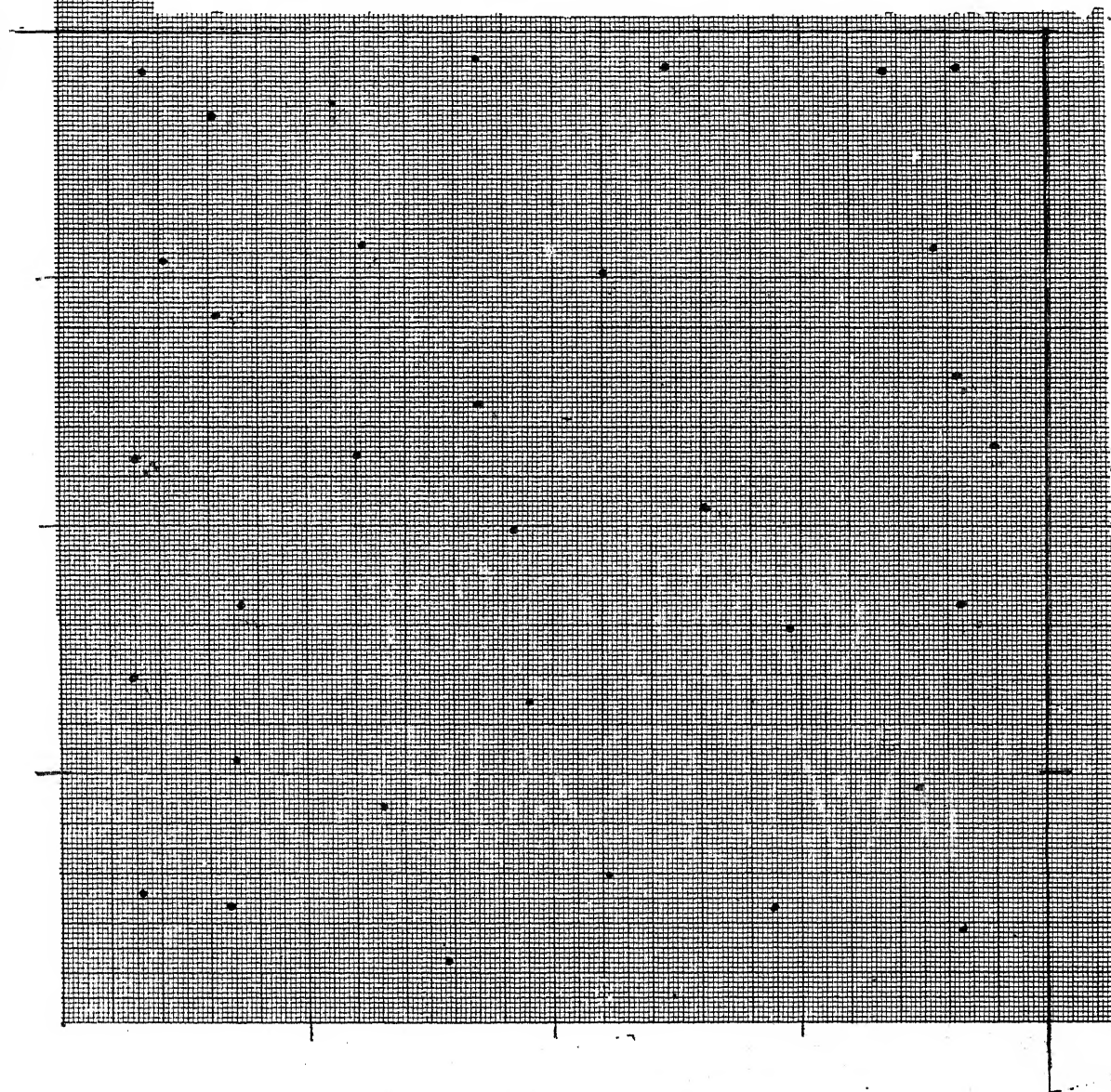
MODEL  $Z=A+BX+CY+DX$ 

Z observed	Z fitted
204.5000000	207.2070000
205.3000000	209.2020000
204.2000000	209.9230000
205.4000000	208.9430000
204.1000000	209.5870000
205.3000000	209.5170000
205.3000000	207.4410000
205.3000000	209.6150000
204.4000000	209.9230000
204.0000000	209.1530000

Posterior Variance = 4.3203000

Fig 6.3 LOCATIONS OF DATA POINTS FOR UNDULATED TERRAIN

SCALE 1 cm = 20 mts.



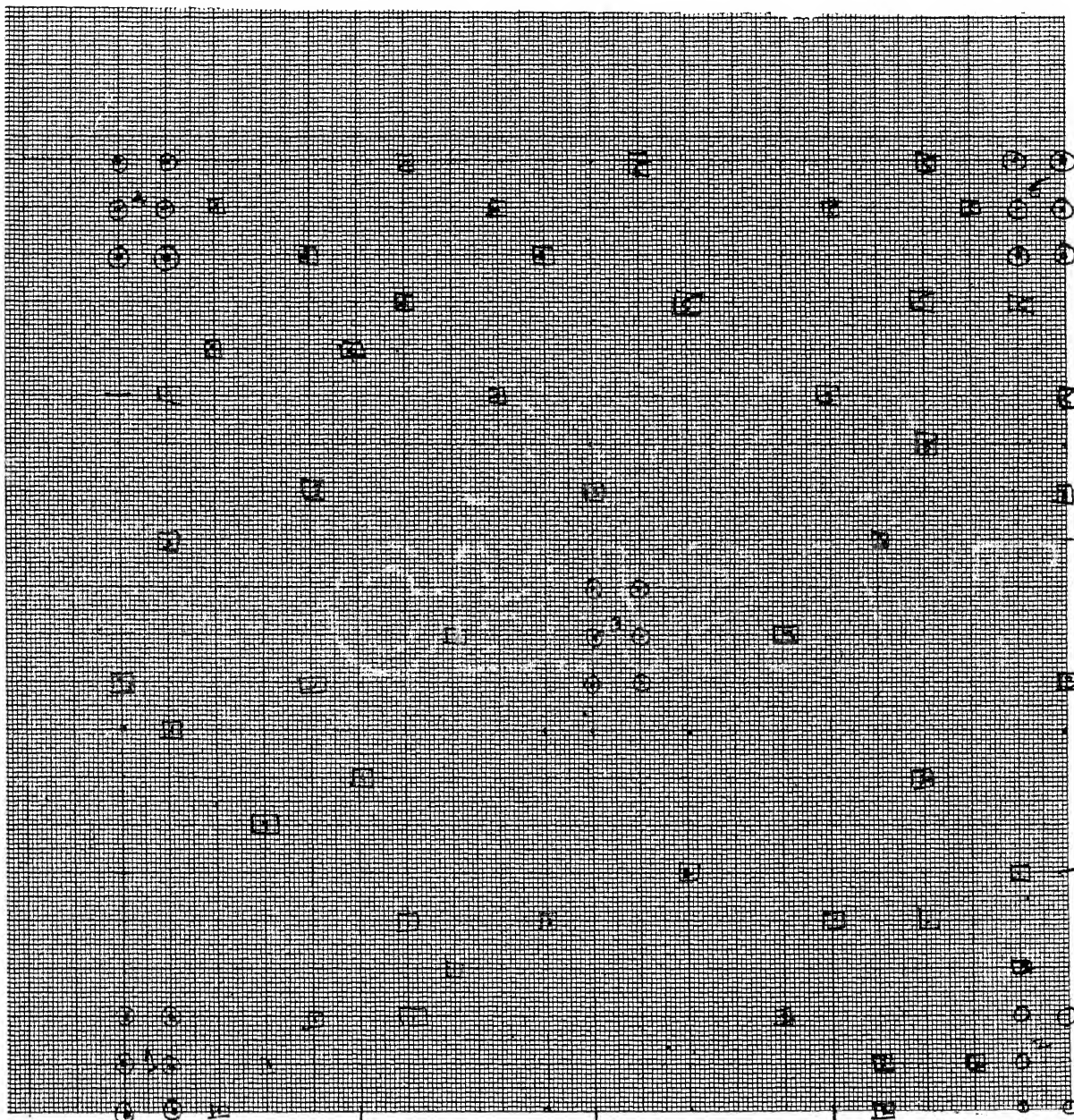
INPUT DATA FOR UNDULATED TERRAIN  
 \*\*\*\*\*  
 TABLE 6.1.3.

\*\*\*\*\*

31.5	51	1985
68.5	48.0	1989
157	22.0	1994
222.5	58.0	1990
288.5	47.5	2002
365.5	48.0	2000
70.5	105.0	2005
130.5	88.0	1987
347.5	92.0	2002
30.5	138.	2000
192	129.	1996
71.5	168.0	1990
294.5	159.0	1982
363.5	168.0	1987.5
185.5	198.0	1994
260	208.5	1990
25	227.5	1997.3
120.5	228.0	1990
170.5	250.0	1990
380	231	2001
362	260	2003
41	306	2000
121	312	1986.4
220	301	2003.5
62	384	2002
353	312	1999
35.5	383	1987.2
61.5	365	1989
110	391	1987
164.5	390	1989
243.5	385	1994.3
332	383	2001
362	386	1997

### 5. LOCATIONS OF POINTS TO TEST MODEL

66



### 6.1.1b Analysis of undulated terrain:

SET I:

The Variance- Covariance matrix for Regression coefficients:

Coeff.	A	B	C	D
A	1.111	-0.056	-0.033	0.002
B	-0.056	0.006	0.002	-1.67E-04
C	-0.033	0.002	0.002	-8.33E-05
D	0.002	-1.67E-04	-8.33E-05	8.33E-06

Correlation matrix for Regression Coefficients:

Coeff.	A	B	C	D
A	1.000	-0.707	-0.775	0.548
B	-0.707	1.000	0.548	-0.775
C	-0.775	0.548	1.000	-0.707
D	0.548	-0.775	-0.707	1.000

Regression Coefficients:

$$A = 1999.333$$

$$B = 0.183$$

$$C = 0.100$$

$$D = 2.23E-09$$

## SET II:

The Variance- Covariance matrix for Regression coefficients:

Coeff.	A	B	C	D
A	6.267	-0.313	-0.188	0.009
B	-0.313	0.031	0.009	-9.40E-04
C	-0.188	0.009	0.009	-4.70E-04
D	0.009	-9.40E-04	-4.70E-04	4.70E-05

Correlation matrix for Regression Coefficients:

Coeff.	A	B	C	D
A	1.000	-0.707	-0.775	0.548
B	-0.707	1.000	0.548	-0.775
C	-0.775	0.548	1.000	-0.707
D	0.548	-0.775	-0.707	1.000

Regression Coefficients:

$$A = 1990.000$$

$$B = -0.204$$

$$C = 2.38E-08$$

$$D = 0.006$$

## SET III:

The Variance- Covariance matrix for Regression coefficients:

Coeff.	A	B	C	D
A	20.851	-1.043	-0.626	0.031
B	-1.043	0.104	0.031	-0.003
C	-0.626	0.031	0.031	-0.002
D	0.031	-0.003	-0.002	1.56E-04

Correlation matrix for Regression Coefficients:

Coeff.	A	B	C	D
A	1.000	-0.707	-0.775	0.548
B	-0.707	1.000	0.548	-0.775
C	-0.775	0.548	1.000	-0.707
D	0.548	-0.775	-0.707	1.000

Regression Coefficients:

$$A = 1992.000$$

$$B = -0.396$$

$$C = 1.19E-07$$

$$D = 0.027$$

## SET IV:

The Variance- Covariance matrix for Regression coefficients:

Coeff.	A	B	C	D
A	22.778	-1.139	-0.683	0.034
B	-1.139	0.114	0.034	-0.003
C	-0.683	0.034	0.034	-0.002
D	0.034	-0.003	-0.002	1.71E-04

Correlation matrix for Regression Coefficients:

Coeff.	A	B	C	D
A	1.000	-0.707	-0.775	0.548
B	-0.707	1.000	0.548	-0.775
C	-0.775	0.548	1.000	-0.707
D	0.548	-0.775	-0.707	1.000

Regression Coefficients:

$$A = 2000.333$$

$$B = 0.233$$

$$C = 0.125$$

$$D = -0.025$$

SET V:

The Variance- Covariance matrix for Regression coefficients:

Coeff.	A	B	C	D
A	9.701	-0.485	-0.291	0.015
B	-0.485	0.049	0.015	-0.001
C	-0.291	0.015	0.015	-7.28E-04
D	0.015	-0.001	-7.28E-04	7.28E-05

Correlation matrix for Regression Coefficients:

Coeff.	A	B	C	D
A	1.000	-0.707	-0.775	0.548
B	-0.707	1.000	0.548	-0.775
C	-0.775	0.548	1.000	-0.707
D	0.548	-0.775	-0.707	1.000

Regression Coefficients:

$$A = 2004.450$$

$$B = -0.039$$

$$C = -0.217$$

$$D = 0.008$$

Table 6.1.4:  $Z = A + BX + CY + DXY$ 

Z observed	Z fitted
1998.00	1999.33
2002.80	2001.33
2012.00	2013.33
2003.40	2003.33
2004.80	2005.33
2008.50	2007.33
2007.00	2007.43
2014.00	2014.90
2010.50	2011.33
2012.80	2013.33

Posterior variance = 1.0657390

Table 6.1.4(a)

Model  $Z = L + MX + NY$ 

Z observed	Z fitted
1998.0000000	2000.3330000
2002.8000000	2004.4330000
2012.0000000	2010.0300000
2003.4000000	2000.4000000
2004.8000000	2006.7960000
2008.5000000	2010.6540000
2007.0000000	2005.4320000
2014.0000000	2016.3330000
2010.5000000	2012.4000000
2012.8000000	2010.7980000

Posterior Varaince = 2.23

Table 6.1.5      MODEL  $Z = A + BX + CY + DXY$ 

Z observed	Z fitted
1973.40	1973.68
1983.90	1983.28
1985.50	1986.64
1996.00	1994.32
1992.10	1993.12
2010.80	2012.08
2006.00	2006.56
2018.00	2017.60
1999.00	1997.92
2020.00	2021.68

Posterior variance = 1.1414550

Table 6.1.5(a)

Model  $Z = L + MX + NY$ 

Z observed	Z fitted
1973.4000000	1975.6440000
1983.9000000	1985.5000000
1985.5000000	1985.4990000
1996.0000000	1993.9200000
1992.1000000	1990.1330000
2010.8000000	2012.8330000
2006.0000000	2004.5350000
2018.0000000	2016.4000000
1999.0000000	2000.1330000
2020.0000000	2018.4950000

Posterior Variance = 1.96

Table 6.1.6

MODEL  $Z = A + BX + CY + DXY$ 

Z observed	Z fitted
1975.90	1976.16
2010.00	2009.28
2168.40	2168.40
2015.90	2015.04
2015.90	2016.48
1969.00	1968.24
1936.80	1936.56
2239.00	2238.96
2153.90	2154.90
2116.10	2115.12

Posterior variance = 0.6879366

Table 6.1.6.(a)

Model  $Z = L + MX + NY$ 

Z observed	Z fitted
1975.9000000	1973.2000000
2010.0000000	2012.5340000
2168.4000000	2170.5330000
2015.9000000	2017.8950000
2015.9000000	2016.9350000
1969.0000000	1971.4950000
1936.8000000	1938.5330000
2239.0000000	2241.5350000
2153.9000000	2156.7950000
2116.1000000	2119.2940000

Posterior Varaince = 2.48

Table 6.1.7

MODEL  $Z = A + BX + CY + DXY$ 

Z observed	Z fitted
1990.90	1992.15
1968.00	1967.49
1940.00	1939.65
1981.50	1981.47
2028.00	2028.29
1929.30	1928.63
1955.00	1954.77
1881.80	1882.61
1761.90	1762.95
1834.00	1833.63

Posterior variance = 0.7013458

Table 6.1.7(a)

Model  $Z = L + MX + NY$ 

Z observed	Z fitted
1990.9000000	1994.8950000
1968.0000000	1964.5350000
1940.0000000	1936.6950000
1981.5000000	1983.4950000
2028.0000000	2030.5330000
1929.3000000	1923.5650000
1955.0000000	1960.4550000
1881.8000000	1879.7990000
1761.9000000	1760.4550000
1834.0000000	1835.2350000

Posterior Varaince = 3.65

Table 6.1.8

MODEL  $Z = A + BX + CY + DXY$ 

Z observed	Z fitted
2001.00	2000.25
2005.00	2004.95
2003.00	2002.11
2011.90	2012.21
2019.00	2018.25
2018.00	2017.89
2052.50	2053.17
2020.90	2021.89
1997.00	1996.83
2130.00	2127.34

Posterior Variance = 1.0832610

Table 6.1.8 (a)

Model  $Z = L + MX + NY$ 

Z observed	Z fitted
2001.0000000	2000.3450000
2005.0000000	2003.4550000
2003.0000000	2001.8980000
2111.9000000	2010.9000000
2019.0000000	2020.7430000
2018.0000000	2023.9980000
2052.5000000	2055.5430000
2020.9000000	2022.9500000
1997.0000000	1998.3330000
2130.0000000	2132.4550000

Posterior Varaince = 2.5

Fig 6.7 ROUTE I

SCALE 1 cm = 20 mts.

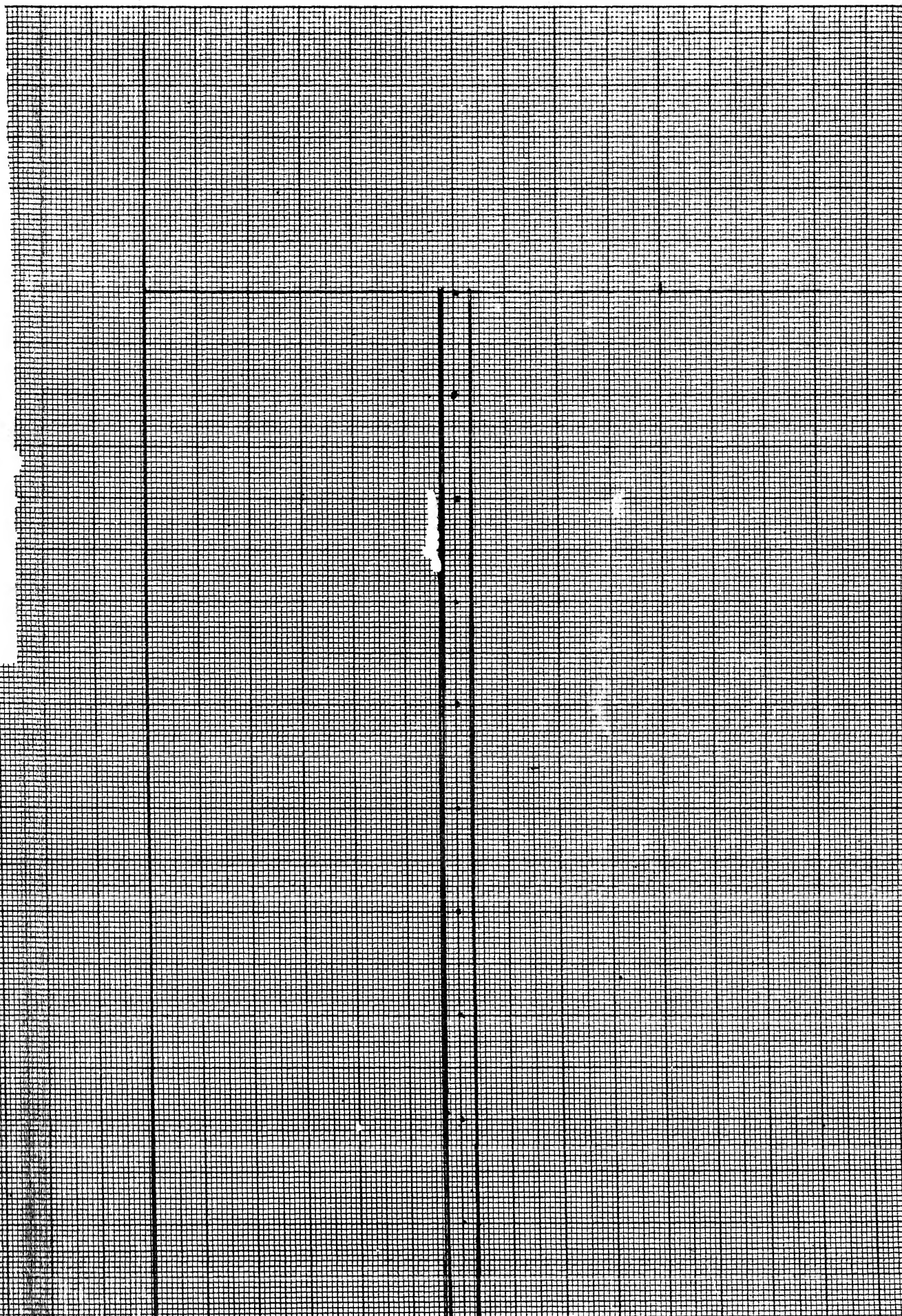


Fig 6.8 ROUTE II

SCALE 1 cm = 20 mts.

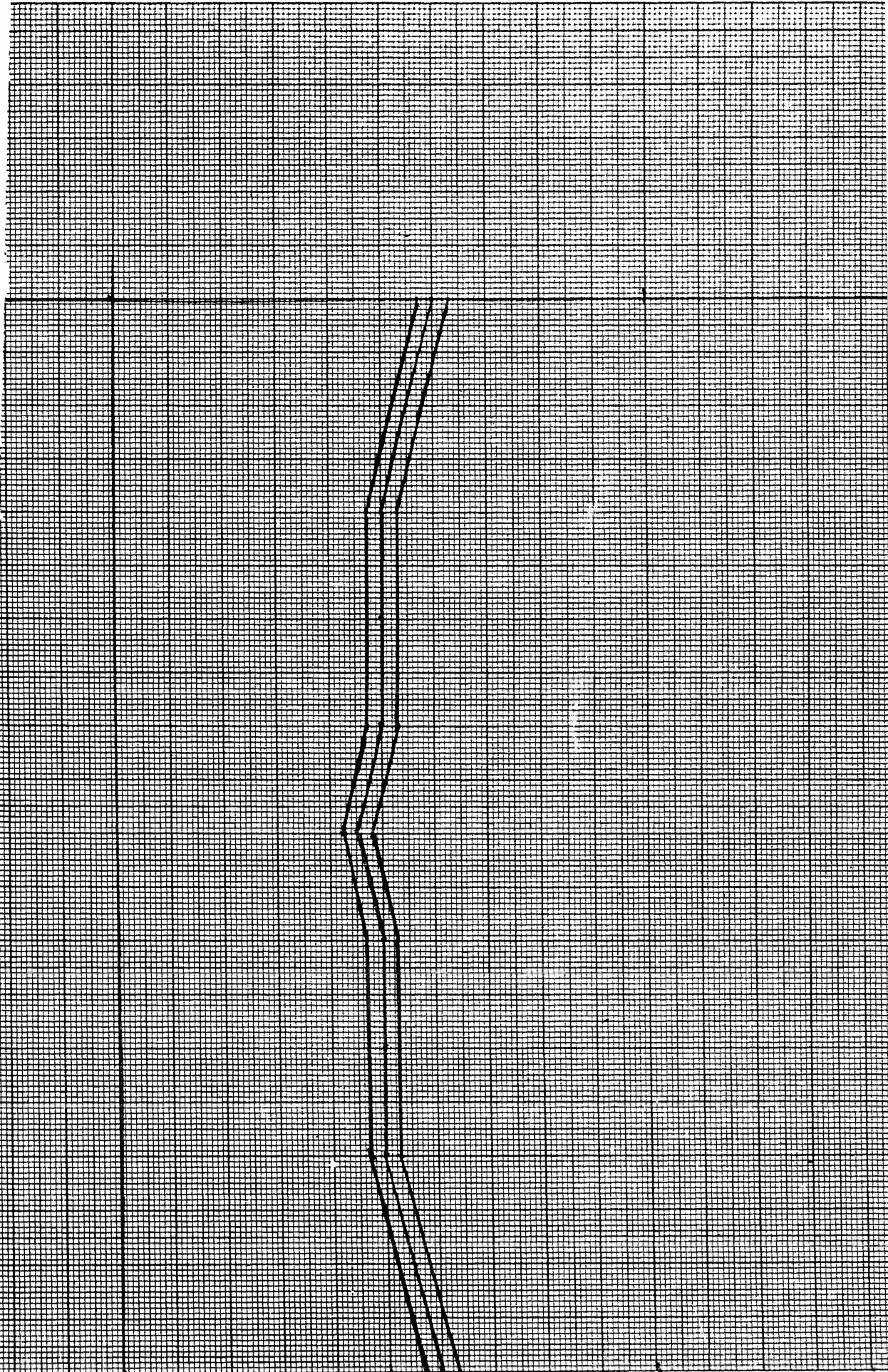
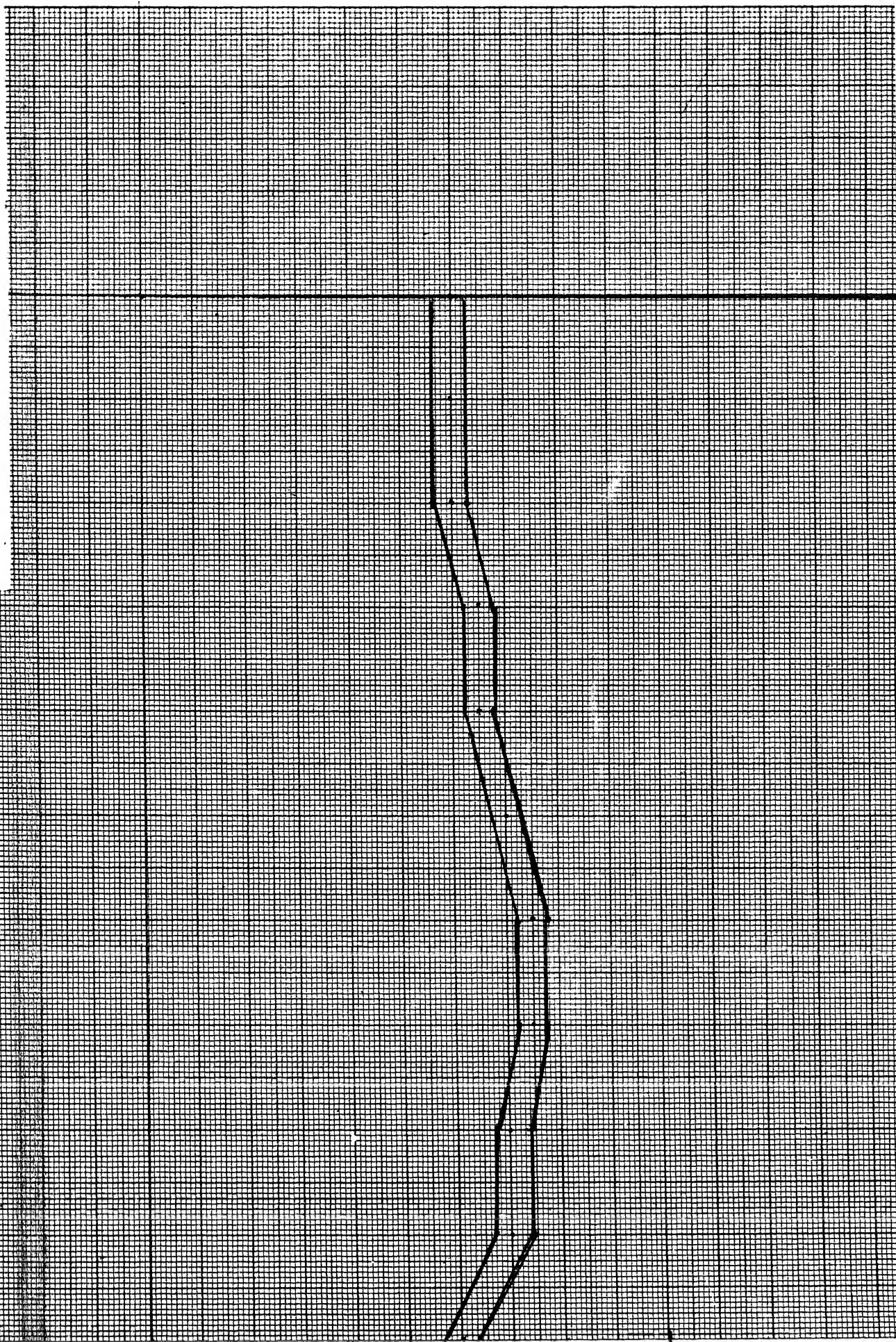


FIG 6.9 ROUTE III

SCALE 1 cm = 20 mts.



ROUTE I:

TABLE 6.1.9:

SECTION NO.	FILL (cubic mts)	CUT (cubic mts)
1		7.178
2.		5.587
3.		3.991
4.		2.393
5.		0.798
6.	0.803	
7.	2.404	
8.	4.003	
9.	5.607	
10.	7.213	

TOTAL VOLUME = 39.174 cubic mts.

ROUTE II:

TABLE 6.1.10:

SECTION NO.	FILL (cubic mts)	CUT (cubic mts)
1.		12.600
2.		6.234
3.		2.243
4.		0.644
5.	3.356	
6.	7.364	
7.	6.56	
8.	5.758	
9.	4.954	
10.	1.748	

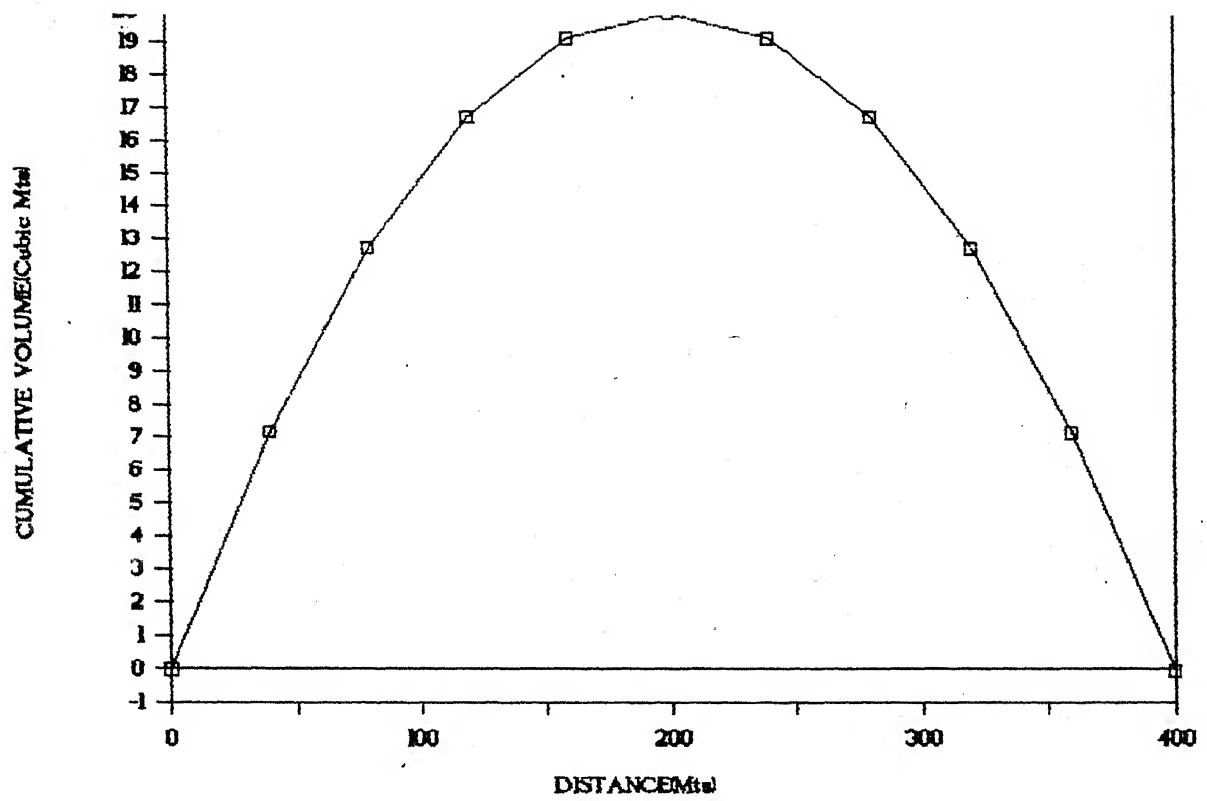
TOTAL VOLUME = 51.36 (cubic mts)

ROUTE III:

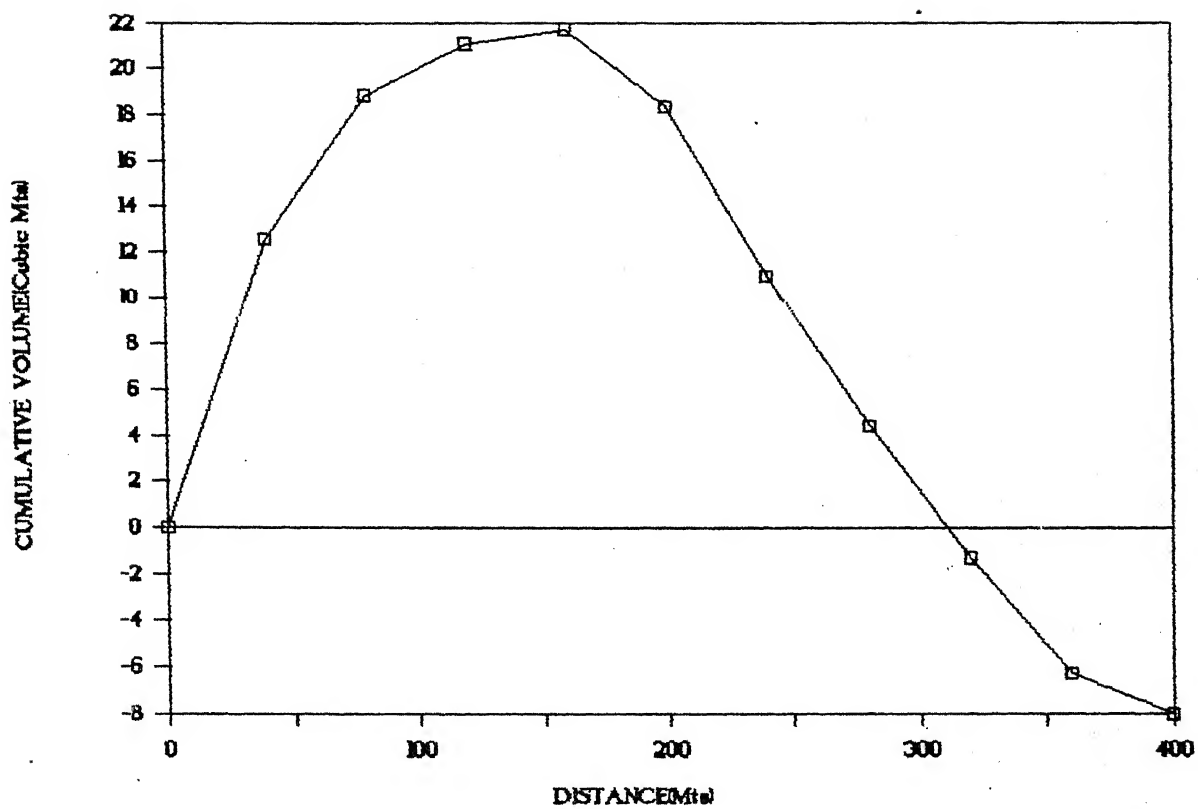
TABLE 6.1.11.

SECTION NO.	FILL (cubic mts)	CUT(cubic mts).
1.		5.871
2.		9.067
3.		9.861
4.		10.656
5.		6.676
6.		0.286
7.	3.717	
8.	7.726	
9.	11.743	
10.	13.354	

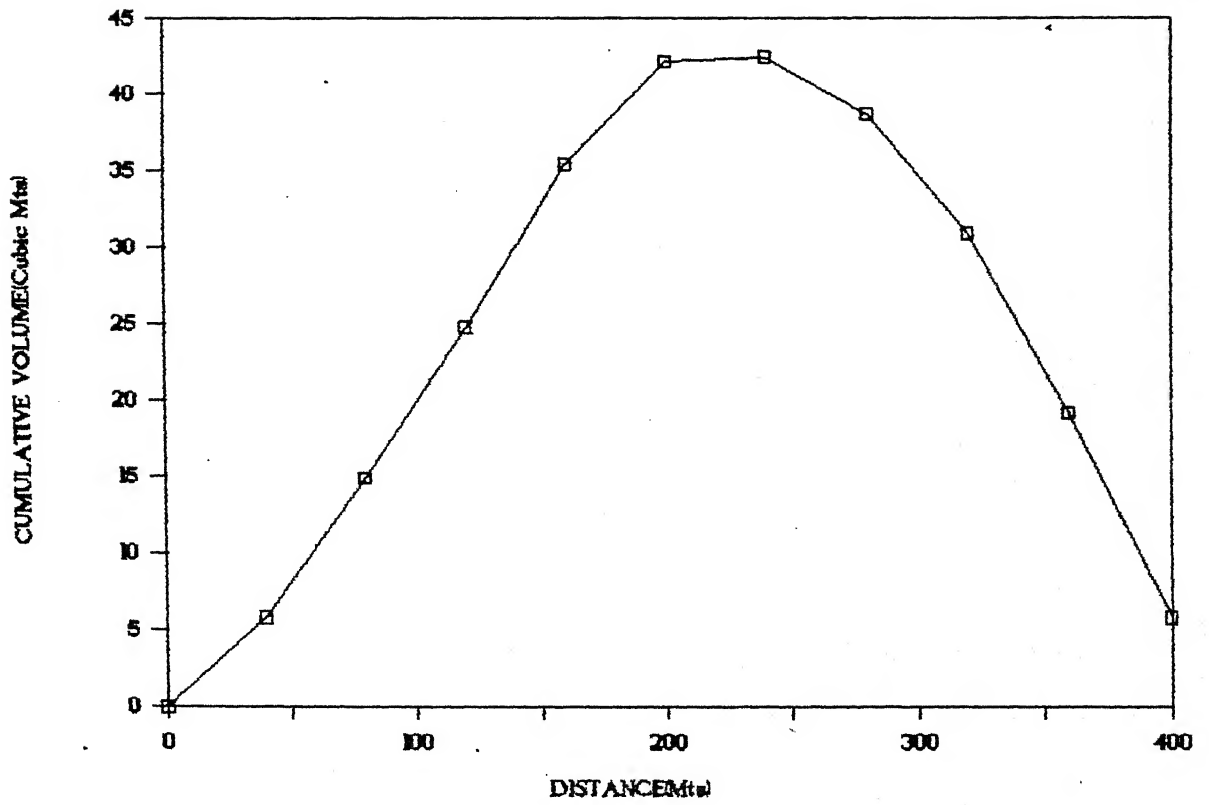
TOTAL VOLUME = 78.914 (cubic mts)



MASS CURVE I



MASS CURVE II

MASS CURVE III

## CHAPTER VII

### CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

In the work presented an attempt has been made to explain the methods to generate the digital terrain model and using it for volumetric computation for highway alignment with the use of digital computers. The following are concluded from the present study.

#### 7.1 Conclusions:

We can generate DTM for truly plane areas with sufficient absolute accuracy ( $\sigma = 0.73m$ ) with the model

$$Z = L + M X + N Y.$$

Similar accuracies may be possible for larger areas and cannot be commented upon since not tested.

WE can generate DTM for undulated terrain by subdividing the area into five parts with the model

$$Z = A + B X + C Y + D X Y.$$

Progressive sampling provides a powerful tool to generate a cartographic database.

Computer graphics is a key element for design and use of

geographical information system. Simulated three dimensional view of terrain shows that graphical product and numerical output are in agreement with each other.

Volumetric computation through DTM provides a powerful tool for alternate alignment selection.

The rapid development in the ability to handle terrain data in a complete digital form holds forth the promise of reducing the drudgery of cartographic operations of providing a wide variety of data interactions, and of reducing time and cost so that managers and decision makers will know how to make maximum utility of resources of world.

DTM provides a base for geographical information system.

## 7.2 Suggestions for future work:

In the present study analysis of flat and undulated terrain are carried out. A suitable mathematical model can be generated for hilly terrain.

For volumetric computation in undulated and hilly areas the length of cross section should be taken at small intervals. Additional corrections such as curvature corrections should be considered.

Spot heights from progressive sampling data can be effectively used for corridor study.

The DTM can be used to rectify satellite born imagery as explained in reference[2].

Contouring can be done by using DTM.

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# APPENDIX

## Geometric Transformations

### Translation

The transformation which translates a point (x,y,z) to a new point (x',y',z') is

$$[x' \ y' \ z' \ 1] = [x \ y \ z \ 1] \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ T_x & T_y & T_z & 1 \end{vmatrix}$$

where  $T_x, T_y$ , and  $T_z$  are components of translation in x, y and z directions respectively.

### Rotation

In three dimensions, it is helpful to devise transformations for rotation about each of the three coordinate axes. Rotation about the Z coordinate axis through angle  $\theta$  is

$$[x' \ y' \ z' \ 1] = [x \ y \ z \ 1] \begin{vmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

The rotation angle  $\theta$  is measured clockwise about the origin when looking at the origin from a point on the +Z axis. Similarly rotation about X and Y axes can be written.

## Scaling

A scaling transformation can be used to scale dimensions in each coordinate directions separately

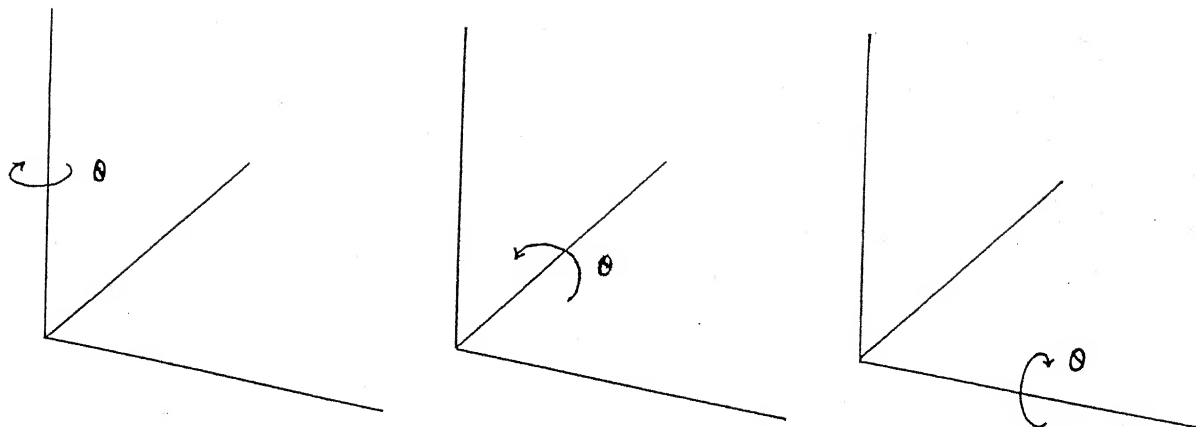
$$\begin{bmatrix} x' & y' & z' & 1 \end{bmatrix} = \begin{bmatrix} x & y & z & 1 \end{bmatrix} \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

## Perspective Transformation

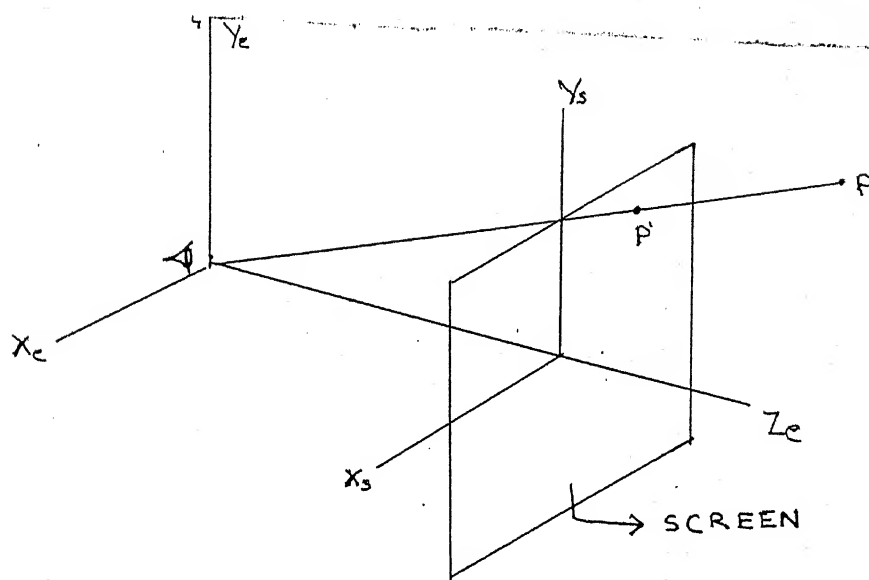
A perspective display can be generated by simply projecting each point of an object onto the plane of display screen as shown fig. The coordinates  $(X_s, Y_s)$  of the projected image of point P measured in eye coordinates  $(X_e, Y_e, Z_e)$  are

$$X_s = (D \cdot X_e / S \cdot Z_e)$$

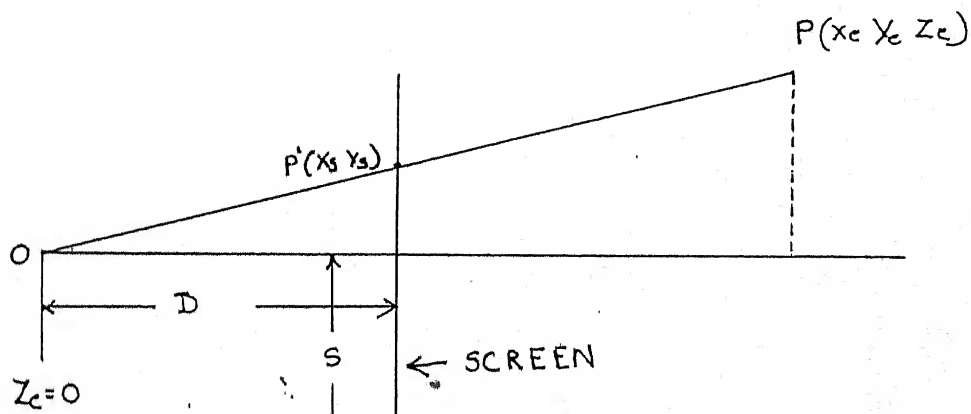
$$Y_s = (D \cdot Y_e / S \cdot Z_e)$$



THE THREE PRIMITIVE THREE DIMENSIONAL ROTATIONS



THE PERSPECTIVE PROJECTION OF POINT P ONTO DISPLAY SCREEN



THE  $(X_s, Z_s)$  PLANE SHOWING DETAILS OF THE PERSPECTIVE PROJECTION

```
C
C C C C C
PROGRAMME TO CONVERT A DISCRETE DATA IN GRIDDED PATTERN
***** BY D.V.NAGARAJU *****
DECLARATION
REAL A(100,3)
REAL B(250,250)
REAL MINX,MINY,MAXX,MAXY,DELTAX,DELTAY
REAL X,Y,SMALL
REAL DIST(100)
INTEGER MAXPS,MAXRS,MAXCS
OPEN
OPEN(UNIT=21,FILE='UNIN.DAT')
OPEN(UNIT=24,FILE='UNOUT.DAT')
OPEN(UNIT=23,FILE='ELE.DAT')
READ STATEMENTS
READ(21,*) MAXPS,MAXRS,MAXCS
READ(21,*) MINX,MINY,MAXX,MAXY,SMALL
READ(21,*)((A(I,J),J=1,3),I=1,MAXPS)
      DELTAX=(MAXX-MINX)/(MAXCS-1)
      DELTAY=(MAXY-MINY)/(MAXRS-1)

MAIN LOOP
WRITE (24,80)
FORMAT(18X,'X',9X,'Y',7X,'Z'///)

Y = MINY
DO 100 I = 1,MAXRS
  X=MINX
DO 101 J=1,MAXCS
  CALCULATE DIST** BETWEEN CURRENT GRIDPOINT AND ALL DATA POINTS
  DO 102 K =1,MAXPS
    D = (X-A(K,1))**2+(Y-A(K,2))**2
    DIST(K)=SQRT(D)
  CONTINUE
  FIND THE NEAREST 4DATA POINTS AND CALULATE SUMS
  S1=0.0
  S2=0.0
  DO 103 K=1,4
    IC=1
    DO 104 L=2,MAXPS
      IF(DIST(L).LT.DIST(IC)) IC=L
    CONTINUE
    IF (DIST(IC).LT.SMALL) GO TO 10
    S1=S1+A(IC,3)/DIST(IC)
    S2=S2+1.0/DIST(IC)
    DIST(IC) = 9999999.99
  CONTINUE
  CALCULATE GRID POINT AND STORE IN MATRIX
  Z=S1/S2
  Z=A(IC,3)
  WRITE(23,*) Z
  IF(J.EQ.1) WRITE(24,81) I,X,Y,Z
  IF(J.NE.1) WRITE(24,82) X,Y,Z
  IF(J.EQ.67) WRITE(24,83)
  X=X+DELTAX
101 CONTINUE
  Y=Y+DELTAY
CONTINUE
100 FORMAT(10X,I2,2X,F8.3,3X,F6.1,3X,F6.1)
81 FORMAT(14X,F8.3,3X,F6.1,3X,F6.1)
82 FORMAT(13X,28('-'))
83 STOP
END
```

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```

*****
PROGRAMME TO COMPUTE POSTERIAL VARTENCE
*****
D.V.NAGARAJU
DIMENSION X(10),Y(10),ZO(10),ZF(10),S(10)
REAL A,B,C,D
INTEGER DPTS,L,M,N
REAL SERR,FUN,DEN
OPEN (UNIT =21,FILE = 'INSTAT.DAT')
OPEN (UNIT =24,FILE = 'OUST.DAT')
READ (21,*) DPTS
READ (21,*) (X(I), I=1,DPTS)
READ (21,*) (Y(I), I=1,DPTS)
READ (21,*) (ZO(I),I=1,DPTS)
DEN =DPTS-1
MAIN PROGRAMME
DO 77 I=1,DPTS
ZF(I) =A+B*X(I)+C*Y(I)+D*X(I)*Y(I)
CONTINUE
DO 88 I = 1,DPTS
S(I) =(ZO(I)-ZF(I))**2
CONTINUE
SS =0.
DO 99 I = I,DPTS
SS =SS +S(I)
CONTINUE
FUN =SS/DEN
SERR =SQRT(FUN)
WRITE (24,30) (ZO(I),I=1,DPTS)
WRITE (24,30) (ZF(I),I=1,DPTS)
FORMAT (5X, 3F10.8)

WRITE (24,50) SERR
FORMAT (15X, F10.2)
STOP
END

```

CCCCC

\*\*\*\*\*  
 PROGRAMME TO PERFORM PROGRESSIVE SAMPLING CELL(GRID)WISE  
 THE SUBROUTINES USED ARE RPROC,CPROC,PATPRO  
 \*\*\*\*\*  
 D.V.NAGARAJU

REAL INMAT(30,35),I,M,N  
 REAL PATMAT(33,33)

COMMON/A1/PXMIN,PYMIN  
 COMMON/A2/L,M,N,DX,DY,TH  
 REAL ROWBND(33,35)  
 INTEGER MAXRS,MAXCS,PXMIN,PYMIN  
 INTEGER XMIN,YMIN,XMAX,YMAX  
 INTEGER NEWPTS  
 INTEGER PMAXRS,PMAXCS  
 OPEN(UNIT=21,FILE='DATA.DAT')  
 OPEN(UNIT=22,FILE='ELE.DAT')  
 OPEN(UNIT=24,FILE='PAT.DAT')

MAXPTS=33  
 I=208.25  
 M=0.009  
 N=-0.028  
 TH=9.

READ(21,\*)MINX,MAXX,MINY,MAXY  
 READ(21,\*)MAXRS,MAXCS  
 READ(22,\*)((INMAT(I,J),J=1,MAXCS),I=1,MAXRS)

MAIN PROGRAMME  
 DX=(MAXX-MINX)/(MAXCS-1)  
 DY=(MAXY-MINY)/(MAXRS-1)  
 INITIALIZATION  
 INVALID ENTRY IS DENOTED BY INVLD

INVLD=-1  
 PMAXRS=MAXRS-1  
 PMAXRY=MAXCS-1  
 \*\*\*\*\*  
 INITIALIZING ROWBOUNDARY MATRIX FOR INVALID ENTRIES  
 \*\*\*\*\*

DO 10 I=1,PMAXRS  
 DO 10 J=1,PMAXCS  
 ROWBND(I,J)=INVLD

CONTINUE  
 INITIALIZING PATCH MATRIX  
 DO 20 I=1,MAXPTS  
 DO 20 J=1,MAXPTS  
 PATMAT(I,J)=INVLD\*1.

CONTINUE  
 DO 30 I=1,PMAXRS  
 DO 40 J=1,PMAXCS  
 COPENG BOUNDARY INFORMATION FOR PATCH(I,J)  
 DO 50 K=1,MAXPTS  
 PATMAT(I,K) = ROWBND(J,K)

CONTINUE  
 COPENG FROM INPUT MATRIX  
 PXMIN = XMIN+(I-1)\*DX  
 PYMIN = YMIN+(J-1)\*DY  
 X1=(PXMIN+DX/2)  
 Y1=PYMIN  
 X2=PXMIN  
 Y2=PYMIN+DY/2  
 X3=X1  
 Y3=Y2  
 X4=PXMIN +DX  
 Y4=Y2  
 X5=X1  
 Y5=PYMIN+DY  
 PATMAT (1,1) = INMAT(I,J)  
 PATMAT (1,MAXPTS) = INMAT(I,J+1)  
 PATMAT (MAXPTS,1) = INMAT(I+1,J)  
 PATMAT (MAXPTS,MAXPTS) = INMAT(I+1,J+1)  
 PATMAT((MAXPTS+1)/2,1)=L+M\*X1+N\*Y1  
 PATMAT((MAXPTS+1)/2,1)=L+M\*X2+N\*Y2  
 PATMAT((MAXPTS+1)/2,(MAXPTS+1)/2)=L+M\*X3+N\*Y3  
 PATMAT((MAXPTS+1)/2,MAXPTS)=L+M\*X4+N\*Y4  
 PATMAT(MAXPTS,(MAXPTS+1)/2)=L+M\*X5+N\*Y5

CALL PATPRO (PATMAT,MAXPTS)  
 DO 60 IM=1,MAXPTS  
 DO 60 IN=1,MAXPTS  
 WRITE (24,\*) PATMAT(IM,IN)

CONTINUE  
 COPYING LAST COLUMN TO FIRST COLUMN  
 DO 70 IM =1,MAXPTS  
 PATMAT(IM,1)=PATMAT(IM,MAXPTS)

CONTINUE  
 DO 80 IM=1,MAXPTS  
 DO 80 IN=2,MAXPTS  
 PATMAT (IM,IN)=INVLD

80 CONTINUE  
 40 CONTINUE

```

DO 90 IN=1,MAXPTS
PATMAT (1,IN)=INVLD
90 CONTINUE
30 CONTINUE
STOP
END

C *****
C SUBROUTINE TO PROCESS A PATCH
C *****
SUBROUTINE PATPRC(PATMAT,MAXPTS)
DIMENSION PATMAT(33,33)
INTEGER RUNNO
REAL L,M,N
COMMON/A2/L,M,N,DX,DY,TH
DO 43095 I=1,4
RUNNO=RUNNO+I*ARZN
NEWPTS=0
100 IF (NEWPTS.EQ.0) GO TO 150
101 IF (RUNNO.EQ.4) GO TO 150
NEWPTS=0
DO 110 I=1,MAXPTS
IQ=I+1
110 CALL RPROC (IQ,RUNNO,PATMAT,MAXPTS)
CONTINUE
120 DO 120 J=1,MAXPTS
IQ=J+1
120 CALL CPROC(IQ,RUNNO,PATMAT,MAXPTS)
CONTINUE
GO TO 100
GO TO 101
43095 TYPE*,RUNNO
CONTINUE
150 RETURN
END

C *****
C SUBROUTINE TO PROCESS ROWS IN PATMAT
C *****
SUBROUTINE RPROC(ROWNO,RUNNO,PATMAT,MAXPTS)
DIMENSION PATMAT(33,33)
INTEGER RUNNO,ROWNO
REAL L,M,N
COMMON/A1/PXMIN,PYMIN
COMMON/A2/L,M,N,DX,DY,TH
OFFSET=(MAXPTS+1)/2
DO 200 I=1,RUNNO
OFFSFI=(OFFSET+1)/2
200 CONTINUE
DO 210 J=1,MAXPTS-2
IF (PATMAT(ROWNO,J).EQ. INVLD) GO TO 210
IF (PATMAT(ROWNO,J+1).EQ. INVLD) GO TO 210
IF (PATMAT(ROWNO,J+2).EQ. INVLD) GO TO 210
RDF1= PATMAT(ROWNO,J)-PATMAT(ROWNO,J+1)
RDF2= PATMAT(ROWNO,J+1)-PATMAT(ROWNO,J+2)
SDF= RDF1-RDF2
IF (SDF.EQ. TH) GO TO 210
NEWPTS= NEWPTS+ 2
J1CENT= J
J2CENT= J1CENT+1
RX1CEN= PXMIN+ (ROWNO-1)*(DX/MAXPTS)
RX2CEN= RX1CEN
RY1CEN= PYMIN+ (DY/MAXPTS)*(J1CENT-1)
RY2CEN= RY1CEN
IF (PATMAT(ROWNO,J1CENT).EQ. INVLD) GOTO 210
PATMAT(ROWNO,J1CENT)= L+M *RX1CEN +N*(RY1CEN)
210 PATMAT(ROWNO,J2CENT)= L+M *RX2CEN+N*RY2CEN
CONTINUE
RETURN
END

C *****
C SUBROUTINE TO PROCESS COLUMNS IN PATMAT
C *****
SUBROUTINE CPROC(COLNO,RUNNO,PATMAT,MAXPTS)
COMMON/A1/PXMIN,PYMIN
COMMON/A2/L,M,N,DX,DY,TH
DIMENSION PATMAT(33,33)
INTEGER RUNNO
OFFSET=(MAXPTS+1)/2
DO 220 J=1,RUNNO
OFFSFI=(OFFSET+1)/2
220 CONTINUE
DO 230 I=1,(MAXPTS-2)
IF (PATMAT(COLNO,I).EQ. INVLD) GOTO 230
IF (PATMAT(COLNO,I+1).EQ. INVLD) GOTO 230
IF (PATMAT(COLNO,I+2).EQ. INVLD) GOTO 230
CDF1= PATMAT(COLNO,I)-PATMAT(COLNO,I+1)
CDF2= PATMAT(COLNO,I+1)-PATMAT(COLNO,I+2)
CDF= CDF1-CDF2

```

```

IF (CSDP.LE.TH) GO TO 230
NEWPTS =NEWPTS+2
I1CENT =I
I2CENT =I1CENT+1
CX1CEN =PXMIN +(COLND-1)*(DX/MAXPTS)
CX2CEN =CX1CEN
CY1CEN =PYMIN +(DY/MAXPTS)*(I1CENT-1)
CY2CEN =PYMIN +(DY/MAXPTS)*(I2CENT-1)
IF(PATMAT(COLND,I1CENT).EQ.INVLD) GOTO 230
X1C=CX1CEN
X2C=CX2CEN
Y1C=CY1CEN
Y2C=CY2CEN
PATMAT(COLND,I1CEN)=L+M*X1C+N*Y1C
PATMAT(COLND,I2CEN)=L+M*X2C+N*Y2C
CONTINUE
RETURN
END

```

230

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C      PROGRAMME TO COMPUTE VOLUME FOR HIGHWAY ALIGNMENT
C      USING CROSS SECTION METHOD
      REAL VOL,CUT,FILL,SCUT,SFIL,SS,W,ZPDEM,VIJ,CSL
      INTEGER L,M,N
      DIMENSION X(20),Y(20),ZDTM(20),ZCOR(20),AREA(20),VOL(20)
      OPEN (UNIT =21,FILE='ROUTE1.IN')
      OPEN (UNIT=24,FILE='ROUTE1.OUT')
      READ (21,*) L,M,N
      READ (21,*) SS,W,TPS,CSL
      READ (21,*) (X(I),I=1,TPS)
      READ (21,*) (Y(I),I=1,TPS)
      DO 77 I=1,TPS
77      ZDTM (I) =L+M*X(I)+N*Y(I)
      VIJ =0.
      DO 88 I=1,TPS
88      VIJ =VIJ +ZDTM(I)
      ZPDM = VIJ/TPS
      DO 99 I=1,TPS
99      ZCOR (I) = ZPDM -ZDTM (I)
      DO 109 I =1,TPS
109     AREA (I) = ZCOR(I) *(W+SS*ZCOR(I))
      DO 110 K= 1, TPS-1
110     VOL(K)= 0.5*(AREA(K)+ AREA(K+1))*CSL
      CONTINUE
      WRITE (24,*) (VOL(K),K=1, TPS-1)
      STOP
      END

```